

# Interactions Between the LP Feed and the Shroud and Reflectors

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## Abstract

The possibility of an interaction between the wideband feed and the surrounding mirrors may be studied through an evaluation of the strength of any echo of a sidelobe of the feed from the shroud or the primary or secondary mirror. The Physical Optics approximation allows working out the echo of a signal transmitted as a sidelobe of the feed and returned by one of its surrounding mirrors. For the shroud, the result of the calculation is the first term in an asymptotic expansion of the echo which is proportional to the inverse of the radius of the shroud scaled to the wavelength. For the expected parameters of the shroud, the echo is only about -42 dB, too small to be important. This echo is essentially the geometrical optics reflection of the  $90^\circ$  sidelobe of the feed from the cylindrical shape of the shroud. When the feed is moved to bring the long wavelength phase center to the focal point of the mirror system, a direct reflection is possible for a ray from the high frequency active zone at the tip of the feed and the edge of the secondary. For the parameters of the system this echo is estimated to be about -33 db, which is also probably too small to be a problem. The only other potential problem is an echo of the backlobe of the feed from the bottom edge of the primary.

## 1 Introduction

The ATA antenna system consists of an offset Gregorian mirror arrangement, a log-periodic feed at the secondary focus, and an approximately half cylindrical shroud between the secondary and the bottom of the primary which

surrounds the feed and directs its spillover radiation away from the ground and towards the sky. Figure 1 shows this antenna system. Although the offset arrangement eliminates direct geometrical reflections of the main feed radiation pattern from getting back into the feed, some of the side and backlobes of the feed are reflected back to the feed, particularly from the shroud. The following discussion assesses the magnitude of these echoes. The variation of the feed phase center position with wavelength complicates this issue somewhat. The feed will be mounted on an axial actuator, and it can be moved so that its phase center may be located at the focus of the mirror system at any frequency in the range 1 - 10 GHz. Wherever it may be located, the receiver is always active over the entire range 0.5 - 11 GHz, and standing waves between the receiver and feed and the mirrors and shroud at any frequency in that range could produce instabilities at some frequency that degrades operation at any other frequency. There are three cases that are potentially problematic. The first is the reflection of the spillover side-lobe radiation of the feed from the cylindrical shroud back into the feed. The second is the echo from the secondary mirror of radiation from the tip of the feed when the feed position has been adjusted for in focus operation at a long wavelength. In the third case, there may be an echo of the feed backlobe from the primary mirror at any frequency.

## 2 The Echo from the Shroud

In the calculation, the LP feed axis coincides with the axis of the half cylindrical shroud as shown in Figure 2. This corresponds to the design and should be the worst case since the echo will be focused back onto the feed by the cylindrical shroud. At any frequency the feed radiates from its center of phase, and, by reciprocity, that is the location where it receives at that frequency. Echos returned to other positions on the feed are not detected, only further scattered.

The natural coordinates for the cylinder are  $(\rho, \phi, z)$  as shown in Figure 2, and  $a$  is the radius of the cylinder, also shown. The small arrow shows an element of radiating current at the phase center and indicates the polarization of radiation sent in various directions. The polar coordinate system corresponding to the little vector is  $(R, \theta', \phi')$  with  $R$  and  $\theta'$  shown. A third polar angle system, not shown, is  $(\theta'', \phi'')$ , the system in which the feed gain is known from the EM simulations. Its pole is along the  $z$  axis. These three

systems collapse into one at the end of the calculation and do not need to be separately discussed in much detail. The gain function is approximately symmetric about the z axis, that is , it is independent of  $\phi''$ , and so is  $G(\theta'')$ .

For evaluating the field scattered back from the shroud, a sufficiently accurate approximation finds the field radiated by the current induced in the cylinder due to the feed as if each part of the shroud acts like a section of an infinite metal plane. This is the well known Physical Optics approximation. This current is  $2 \hat{n} \times H_i$ , where  $\hat{n}$  is the local surface normal and  $H_i$  is the incident magnetic field.

With unit input power to the feed, the power arriving at the surface in the solid angle  $d\omega$  is  $G(\theta'')d\Omega$  at the distance R. It is also equal to the pointing vector times the corresponding element of area dA, where  $dA = R^2 d\Omega$ . Since the normal component of the Pointing vector is  $\eta|H|^2/2$ , the following simple relation for the magnitude of the magnetic field at the cylinder follows.

$$\eta|H|^2 = 2G(\theta'')R^{-2} \quad (1)$$

In the sidelobes of the pattern, the effective radiating size of the antenna is of order  $\lambda/2$ , so that the cylinder is in the far zone of the feed. Then the polarization of the magnetic field and its phase can be read from Fig 2, giving the following formula for the field at the cylinder.

$$H_{\phi'} = \sqrt{\left(\frac{2G(\theta'')}{\eta R^2}\right)} \sin(\theta') e^{-ikR} \quad (2)$$

where  $k = 2\pi/\lambda$ .

Only the  $\phi$  component of the induced surface current can radiate a field component that can be received by the feed with its indicated polarization. This has the form  $J_\phi = 2H_{\phi'} f$ , where f is the vector product, a function of the angles that can be evaluated later. An increment of the back radiated electric field that has the same polarization as the feed vector is

$$dE_f = J_\phi \frac{\sin(\theta') i \omega \mu e^{-ikR}}{4\pi R} dz d\phi \quad (3)$$

Substituting (2) into (3) and integrating the current distribution over the extent of the shroud,  $\pi/2 \leq \phi \leq 3\pi/2$  and  $-1 \leq z \leq 1$ , leads to the following formula for the electric field radiated back to the feed.

$$E_f = \int_{\pi/2}^{3\pi/2} \int_{-l}^l \frac{2\sin^2(\theta') f i \omega \mu}{4\pi R^2} (2G/\eta)^{1/2} e^{-2ikR} a d\phi dz \quad (4)$$

The integration over  $z$  should be done first because there is a stationary point in the phase of the integral. This allows an asymptotic approximation for this type of integral to be used (Morse & Feshbach, 1953, Methods of Theoretical Physics, Part 1, p440). Given an integral of the form

$$I = \int_a^b g(t) e^{ixh(t)} dt \quad (5)$$

If  $h'(\tau) = 0$ , then the first term in an asymptotic expansion in inverse powers of  $x$  is

$$I \sim \left( \frac{2\pi}{x|h''(\tau)|} \right)^{1/2} g(\tau) e^{ixh(\tau)} e^{is\pi/4}, \quad (6)$$

where  $s$  is the sign of  $h''(\tau)$ . In the present case, where  $R^2 = a^2 + z^2$ ,  $h(z) = (1 + z^2/a^2)^{1/2}$  and  $x = 2ka$ , and the stationary point is at  $z=0$ . This is the expected result that the principal echo is in the plane where the echo is specular. Everything simplifies in this plane where  $z=0$ ,  $f=1$ ,  $\theta' = \phi$ ,  $h''(0) = a^{-2}$ , and  $G(\theta'') = G(\theta'' = 90^\circ)$ . The integral of  $\sin^2(\phi)$  is just  $\pi/2$ . With these substitutions, the integral becomes

$$E_f \sim \frac{i\omega\mu}{4a} e^{i(\pi/4 - 2ka)} \left( \frac{2G\pi a}{\eta k} \right)^{1/2} \quad (7)$$

The power received back at the feed is  $\frac{|E_f|^2}{2\eta} A_e$ , where  $A_e$  is the effective receiving area,  $\frac{G\lambda^2}{4\pi}$ . Substitution and simplification leads to the final formula for the coupling of the feed to itself due to the roundtrip echo from the shroud.

$$P_r/P_i = \frac{\pi G^2(90^\circ)}{32} (\lambda/a) \quad (8)$$

$G(90^\circ) = .035$  (-14.5 dB). The longest wavelength is 0.60m, and the radius of the shroud is about 1.2m. Entering these numbers gives a round trip coupling of only -42 db, with less at the shorter wavelengths.

### **3 Reflections from the Secondary**

If the feed is moved forward in order to bring the phase center of a longer wavelength into focus, the tip of the feed (active at the high frequencies) will see a direct echo from the edge of the feed. The ray from the focal point that goes to the top of the primary after a reflection from the edge of the secondary is the important ray in this case. As the feed is moved forward to bring the feed phase center at longer wavelengths to the focal point, the tip of the feed radiates a ray to the edge of the secondary which reflects closer to the tip rather than going to the edge of the primary. At a deflection of 40 cm, the ray from the tip reflects back to the tip. Since the curvature of the secondary is similar to that of the shroud, equation (8) may be used to get an estimate of the strength of the reflected signal in this case. The feed gain now corresponds to the angle near the edge of the secondary, about  $49^\circ$ . At this angle the feed gain is about -3.5 dB. Also, the distance will be larger, about 1.5m, and the wavelength 3cm for the top of the band. In this case the estimated strength of the echo is -33 dB. This is small enough that no amplifier instabilities should result. A 40cm motion of the feed is required to change the focus from about 6 GHz to 1 GHz, and it is the maximum planned motion for the feed.

### **4 Reflection of the Feed Back Lobe by the Primary Mirror**

If the bottom edge of the primary is behind the feed in such a way that a specular reflection of the feed's backlobe returns to the feed point, a round trip echo of about -17 dB could result. This is large enough that the antenna input impedance would be significantly affected. To avoid this problem, the back-end of the feed needs to be moved away from the primary mirror. If that is not possible for reasons of mechanical stability, then a tapered wedge which can spoil the specular reflection should be located behind the feed.

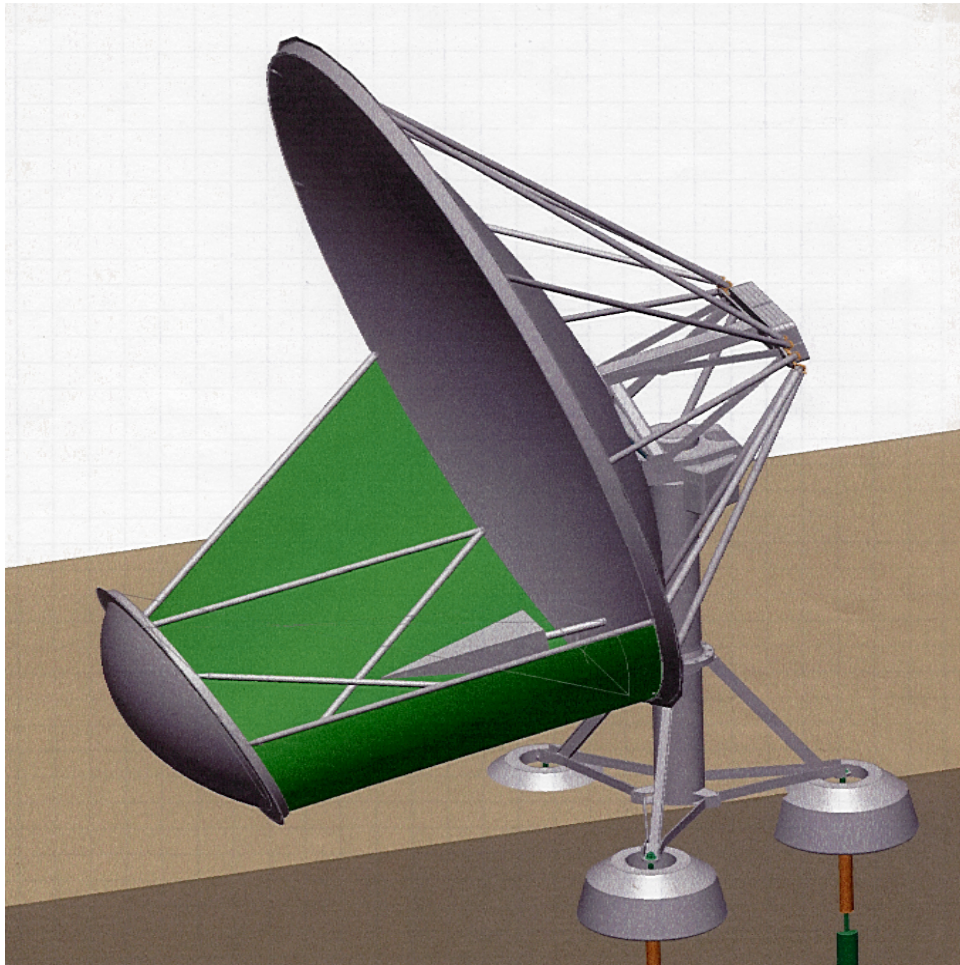


Figure 1: The offset gregorian optical system. The pyramid is the feed. The shroud surrounds the feed in the space between the two reflectors.

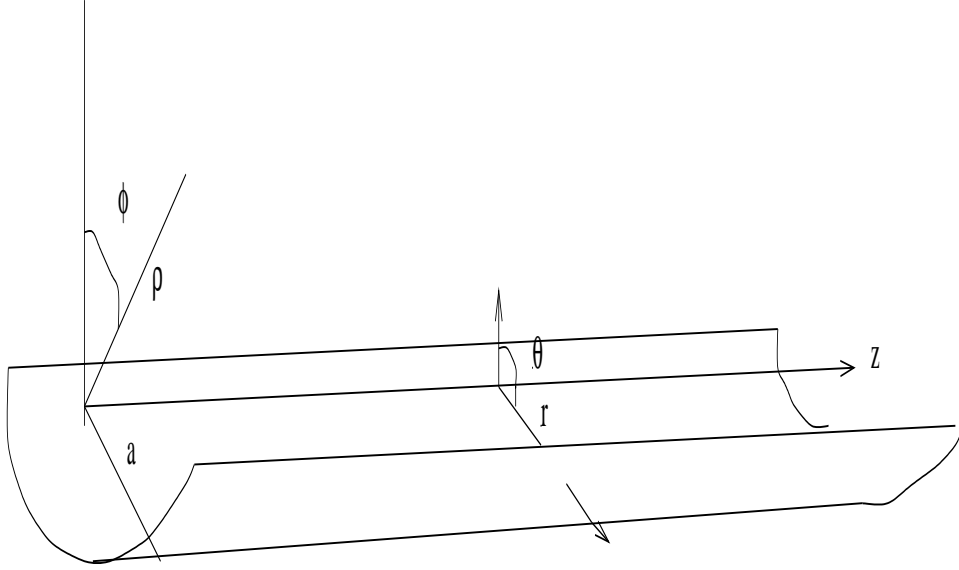


Figure 2: The shroud is represented by the half cylinder. The radiating part of the feed is shown as the vertical arrow in the center. The coordinates correspond to those in the text. The feed arrow is in the plane  $z=0$ .