Expected Properties of the ATA Antennas

Jack Welch and Dave DeBoer

June 21, 2004

Abstract

A simple form for the ATA antenna pattern follows from the aperture field given by the feed pattern. The corresponding gain pattern can be accurately fit by a Gaussian with the same FWHM. To use the Gaussian as the pattern requires defining a beam efficiency which includes the effects of the feed pattern which dominates the sidelobes at wide angles and the effects of mirror edge diffraction. A further efficiency factor is defined which includes the effects of transmission losses and Ruze losses. A simple formula is found for observing the moon using the Gaussian approximation. It gives the expected antenna temperature and the observed fraction of the moon's flux as a function of the antenna beam diameter. Based on the efficiency factors, the expected aperture efficiencies are found to be .54 at 500 MHz,.58, at 1.5 Ghz, and.54, at 10 Ghz. The beam efficiencies at these three frequencies are .78, .79, and .76, respectively. The expected system temperature as a function of frequency is also discussed. Temperatures in the range of 30K (at 1.5 GHz) to 40K (at 10 Ghz) are expected. Finally, a correction is worked out which gives the factor by which the observed antenna temperature of an extended source must be multiplied to emulate an unresolved measurement of the extended source.

1 The Antenna Pattern

The pattern is dictated by the size of the antenna aperture (6.1 m), the pattern of the feed, and the feed amplitude at the aperture edge. The feed has as much gain as we can have for a simple log-periodic antenna, 12 dB, and the -13 dB edge illumination is at an angle of 42^o with respect to the feed axis. The -13dB is chosen as a compromise between sidelobe level and overall antenna gain. This low edge illumination is also chosen to minimize the edge diffraction losses for this low frequency antenna. The feed gain is -1dBi at the edge of the mirror system. The 42^o angle fixes the focal ratio of the optics to be 0.65. Unfolding the Gregorian optics gives us an equivalent symmetric prime focus 6.1 m antenna with a focal length of 3.97 m. The feed pattern mapped on to the aperture provides an approximately symmetric linearly polarized electric field which is fit pretty accurately by the following function of radius.

$$E(r) \propto .211 + .789[1 - (r/3)^2]^{1.9} volts/m \tag{1}$$

The form of the expression is chosen so that the Fourier Transform for the distant field has a simple analytic form.

$$E(\theta) \propto J_1[(6\pi/\lambda)Sin[\theta]]/((6\pi/\lambda)Sin[\theta]) \tag{2}$$

$$+25.40 * J_{2.9}[(6\pi/\lambda)Sin[\theta]]/((6\pi/\lambda)Sin[\theta])^{2.9} volts/m \tag{3}$$

The aperture radius is 3 m, θ is the polar angle from the pattern central axis, and λ is the wavelength in meters. The pattern is confined to angles close to the axis, and so the polarization in the radiated field is close to that of the aperture field. The other polarization has a similar field distribution.

The gain function is proportional to the square of $E(\theta)$ and is plotted in Figure 1 for a wavelength of 0.2 m, 1.5 GHz. The log-periodic feed pattern is frequency independent and illuminates the optics similarly at all wavelengths. With this illumination, the angular width of the pattern scales directly with the wavelength. From the pattern of Figure 1, we find the following FWHM:

$$\Theta_{1/2} = 2.33^o(\lambda/0.2m) \tag{4}$$

A more useful form for the pattern for analyzing maps made with this antenna can be found by approximating it with a symmetric Gaussian. This Gaussian, chosen to have the same FWHM and normalized to a peak value of 1, is also plotted in Figure 1. It is the upper curve. The curves are remarkably similar. A log plot of the gain curve, based on Equation (2,3), shows ever weakening sidelobes as the angle increases out to 42^o. Beyond this angle, the feed itself looks past the edge of the secondary mirror, and its radiation pattern dominates the sidelobes at larger angles. For most mapping, the astronomical sources are generally much smaller than 42^o, and one can use the Gaussian approximation to the gain curve to analyze them. This requires, however, some care with the normalization of the Gaussian.

There are three parts to this normalization. Overall, the gain pattern divided by 4π must integrate to unity over the whole sky, 4π steradians. The first part of this normalization is to compare the integral of the calculated pattern and the integral of the Gaussian approximation over the polar angle, θ, out to a polar angle of about 42^o or less (including 2π for the axial angle). We find that the Gaussian as defined (with same FWHM as the calculated pattern) has 7% larger integrated value than the same integral over the calculated gain pattern. The second part is the inclusion of the true pattern over the rest of the solid angle beyond $\theta = 42^o$. The Gaussian rapidly approaches zero asymptotically there. This part of the pattern is dominated by the feed radiation. The feed radiates directly to higher elevations and indirectly there due to scattering of the lower angle radiation by the shroud. In detail this radiation pattern is very complicated. However, all we really want is the integrated total, and we can get that from the feed pattern by itself. The simulations show that the total radiation beyond 42^o is 18%. The third correction comes from the edge diffraction of the two reflectors. This effect has been studied by Kildal (1983, IEEE, PGAP, AP-31). It is a small wavelength dependent factor, and, for our geometry, amounts to $(1 - .095\sqrt{\lambda(m)})$. We adopt the Gaussian function for the gain and scale its usual normalization by the additional term (with its three factors defined above):

$$\eta_B = (1.07)(.82)(1 - .095\sqrt{\lambda(m)}) \tag{5}$$

η_B is sometimes called the "beam efficiency", not to be confused with the "aperture efficiency". The Gaussian gain function which we use to describe integrations over sources that do not extend more than about 40^o is:

$$G_g(\theta,\phi)/4\pi = \eta_B \frac{.879}{(\Theta_{1/2})^2} \times Exp[-2.76\frac{(\theta)^2}{(\Theta_{1/2})^2}] \tag{6}$$

The sub script "g" distinguishes the approximate gain function from the true gain function which integrates to unity. In summary, the integral of the gain function of equation (6) gives the following result.

$$\int_0^{2\pi}\int_0^{\pi} \frac{G_g(\theta,\phi)}{4\pi} Sin[\theta] d\theta d\phi = \eta_B \tag{7}$$

The importance of this normalization may be more clear in the later discussion of observations with the telescope.

2 Correction for Input Losses

Losses in the transmission line between the feed and the LNA reduce the signals received by the antenna as well as contributing to the system temperature by noise emission. We define the input terminals to be at the input to the LNA. At this location, the effective antenna gain and collecting area are lower due to these losses and we introduce a loss efficiency factor, η_l, to account for them. There are several contributions: (1) Loss in the feed itself is estimated to be about 3% independent of frequency. (2) The tip circuit board contributes 1%. (3) A 1.7 inch ambient temperature two wire line below the tip circuit is 1%. (4) A two inch line over which the temperature changes from 300K to 80k is 1.3%. (5) The 4.9 inch glass balun at 80K is estimated to have a loss of 4.0%. The losses for items (2) through (5) are for 10 GHz and scale as $\sqrt{f/10}$. Finally the rough surface Ruze loss of the primary mirror produces a gain loss factor of $Exp[-(.798/\lambda(cm))^2]$ due to a surface RMS of about 0.6mm. Putting all these together gives the following total correction.

$$\eta_l = [1 - (.03 + .073\sqrt{f/10})]Exp[-(.798/\lambda(cm))^2] \tag{8}$$

Typical values are $\eta_l(10GHz) = 0.81$, and $\eta_l(1.5Ghz) = .94$. The emission contribution of these components to the system temperature is about 20K. The Ruze loss has no extra emission contribution, because the extended sky background emission should be about the same into either the normal sidelobes or the rough surface scattered sidelobes. η_l is an additional factor that should multiply the gain function of equation(6). This gives a total beam efficiency $\eta_{BT} = \eta_B\eta_l$. η_{BT} is plotted in figure 2 as a function of frequency.

3 Observing Radio Sources

Every antenna is characterized by a gain function for each polarization, $G(\theta, \phi)$. In general, this function divided by 4π integrates to unity over the whole sky, as noted above. If the antenna input power is P_i, then $P_iG(\theta, \phi)/4\pi$ is the power radiated per unit solid angle. The antenna is also characterized by an effective receiving area in every direction, $A_e(\theta, \phi)$. These two functions have a simple relationship that expresses reciprocity.

$$G(\theta, \phi) = \frac{4\pi A_e(\theta, \phi)}{\lambda^2} \tag{9}$$

The sky at any wavelength is characterized by a brightness distribution, $B(\theta, \phi)$. We can speak about a sky brightness temperature distribution by using the Planck function for B. At radio wavelengths the Raleigh-Jeans approximation usually applies with B simply proportional to T. For each polarization, $B(\theta, \phi) = kT(\theta, \phi)/\lambda^2$. Another important quantity is the flux of the radiation defined for different pointing directions.

$$F = \int_0^{2\pi} \int_0^{\pi} B(\theta, \phi) d\Omega \qquad (10)$$

This is the power per unit area with the normal to the unit area in the direction $\theta = 0$. We find the total power per unit bandwidth received by our antenna from the sky by weighting the brightness by our area for each direction, $A_e(\theta.\phi)$, and summing over all directions.

$$P = \int_0^{2\pi} \int_0^{\pi} B(\theta, \phi) A_e(\theta, \phi) d\Omega \equiv kT_A \qquad (11)$$

This is for the antenna pointed in the direction of the zenith, and it defines the antenna temperature, T_A, for that direction. The antenna can, of course, be pointed in other directions giving T_A for other directions. Equation (11) can be generalized for this by including the antenna coordinates in the A_e function.

$$kT_A(\theta_a, \phi_a) = \int_0^{2\pi} \int_0^{\pi} B(\theta, \phi) A_e(\theta, \phi; \theta_a, \phi_a) d\Omega \qquad (12)$$

The subscripted coordinates refer to the antenna pointing direction. This is the basic measurement equation. $A_e(;)$ is symmetric between the two sets of variables. It is useful to look at the two limiting cases corresponding to the observation of point sources (unresolved sources) on the one hand and extended sources on the other.

3.1 Point Source Observations

This is the simple case that is most familiar. In equation(12) we assume that we have pointed the antenna at the point source and that the angular extent of the source is small compared to the angular variation of $A_e(\theta, \phi)$. Then $A_e(0)$ factors out of the integral, leaving $A_e(0) \int B d\Omega$, which is just $A_e(0)$F. This is the familiar equation:

$$kT_A = A_e(0)F \qquad (13)$$

k/$A_e(0)$ gives the Janskys per K^o for the antenna per polarization.

3.1.1 Observing an Extended Source

Here we are interested in the brightness distribution of an extended source. Furthermore, let us write the measurement equation (12) in terms of brightness temperature. In the integrand we have $kT_B(\theta, \phi)/\lambda^2$ times $A_e(\theta, \phi)$. We use equation(9) to replace A_e/λ^2 by $G/4\pi$ and cancel the k from both sides of equation (11) to obtain:

$$T_A = \int T_B \frac{G}{4\pi} d\Omega \tag{14}$$

If we put the antenna inside a blackbody, T_B, the blackbody temperature would factor out, and the integrand is exactly one for the proper normalization of the gain function. We get the correct result: T_A=T_B.

Now for an example let us point the antenna directly at the moon. From the discussion above, we know that the Gaussian function is a very good representation of the beam, as long as we normalize it correctly. That is, we must use $\eta_l G_g(\theta, \phi)$. The moon is uniformly bright with temperature T_m=230K (Troitsky, 1960) at all wavelengths longer than about 3 cm. Its angular diameter is θ_m. In equation(14), $T_B = T_m$, a constant, and factors out of the integral. That leaves the following:

$$T_A = T_m \int_0^{2\pi} \int_0^{\theta_m/2} \eta_l \frac{G_g}{4\pi} d\Omega \tag{15}$$

$$= T_m \eta_{BT} 2\pi(.879)/(\Theta_{1/2})^2 \int_0^{\theta_m/2} Exp[-2.76(\theta/\Theta_{1/2})^2] Sin[\theta] d\theta \tag{16}$$

and

$$T_A = T_m \eta_{BT}(1 - Exp[-.69(\theta_m/\Theta_{1/2})^2]) \tag{17}$$

The antenna temperature is a fraction of the Moon's brightness temperature. At the frequency at which the Moon's diameter is equal to the FWHM of the beam, approximately 7 GHz, T_A is half of the Moon's brightness temperature times the efficiency factor. This expression can be used for a measurement of the system temperature. One first points the antenna away from the Moon so that only the total background is detected. This background consists of the receiver temperature, the emission from the losses noted above, and the sky background radiation received by the antenna. It is the system temperature. Then one points the antenna at the moon, and the Moon's contribution given by equation (17) above raises the antenna temperature. Measuring the ratio and knowing the efficiency factors and the Moon's diameter, one can find the system temperature. Apart from the efficiency factor, if the Moon's antenna temperature at 7 GHz equaled the system temperature, the latter would be 115K. Including the efficiency factor of about .76, it would be 87K.

It is also possible to think of the measurement as providing some fraction of the flux of the moon. The Moon's flux is $kT_m\Delta\Omega_m/\lambda^2$. Multiplying both sides of equation (17) by $k\Delta\Omega_m/\lambda^2$ shows that the fraction of the flux that we measure is given by the same factor in brackets in equation (17) as the fraction of the Moon's brightness temperature.

4 The Relation Between Maximum Effective Area and Peak Gain

Equation (9) is the general relation between the gain and effective area in any direction. Writing this for the peak values is especially useful.

$$G(0)/4\pi = A_e(0)/\lambda^2 \tag{18}$$

Also, from equation(6),

$$G(0)/4\pi = \eta_B(.879)/(\Theta_{1/2})^2 \tag{19}$$

For the moment, we leave out the η_l term. Also, to be definite, we put in numbers for 1.5GHz ($\lambda = .20$ m). Here $\Theta_{1/2} = .0406$ radians, from equation (4),and $\eta_B = .84$ from equation(5). From these numbers, we find that $A_e(0) = 17.9m^2$. For a physical area of 29.22 m^2, an aperture efficiency of .61 is implied. This is exactly what we calculated just working out the aperture efficiency. This should be no surprise, since we began both calculations using the same feed pattern. Nevertheless, it is gratifying to the see the agreement. Now we must also apply the loss correction. At 1.5 GHz, it is a factor of .95, which brings the aperture efficiency down to .58. At higher frequencies it is lower, as implied by equation (8). It is .54 at 10 GHz, with a little help from the diffraction term in η_B. This factor must be applied to both G(0) and $A_e(0)$.

5 Measuring $A_e(0)/T_{sys}$ Using the Moon

Pointing the telescope toward the Moon provides the antenna temperature given by equation (17). The ratio of that signal to what is received when the telescope is pointed away from the moon into a slightly different direction is the Y-factor.

$$Y = 1 + T_A/T_{sys} = 1 + \frac{\eta_{BT} T_m}{T_{sys}} \times (1 - Exp[-.69(\theta_m/\theta_{1/2})^2]) \qquad (20)$$

T_m is known as are the moon's diameter,θ_m, and the antenna beamwidth, $\theta_{1/2}$. The fraction, $\frac{\eta_{BT}}{T_{sys}}$, is found from this measurement. Except for the efficiency factors, the effective area of the antenna is pretty well known from standard diffraction calculations. Overall, the effective area in the peak direction, $A_e(0)$, is

$$A_e = 24.0\eta_{BT} m^2 \qquad (21)$$

Multiplying the measured fraction above by 24.0 m^2 gives $\frac{A_e}{T_{sys}}$, the conventional measure of the point source sensitivity of an antenna.

6 The Expected System Temperature

The contributions to the system temperature are from the antenna background noise pickup, the losses in the transmission line into the LNA, and from the LNA itself. The antenna pickup should be dominated by the brightness of the atmosphere with $\leq$ 3K (1%) from the ground. The combination of the emission from the water vapor and oxygen and the CMB is about 6K. The absorption terms in the input lines were discussed above. The total is given by η_l in equation (8). The exponential Ruze loss factor should be omitted, since the pickup in the rough surface scattering should be about the same as it would have been if there were no scattering. We assume that the edge diffraction losses couple half to the ground and half to the atmosphere. For those loss terms in room temperature structures, the emission is just the loss multiplied by about 300K. This includes the feed, the tip circuit, and the ambient transmission line. The loss of the balun is multiplied by the dewar temperature of 80K. In the section of line over which the temperature rises from 80 K to 300 K, the integral $\int T(z)\alpha(z)dz$ must be calculated. Two frequency dependent terms emerge. The resistive losses in the transmission lines scale with the square root of the frequency. The edge diffraction losses scale with square root of the wavelength. For the LNA we take a constant value of 10K, although the model result based on measurements predicts a minimum at about 8K in the middle of the 0.5 - 12 GHz band (Wadefalk, private communication). The sum of all these contributions is:

$$T_{sys} = 7.8 * f^{-1/2} + 6.6 * f^{1/2} + 16.K \tag{22}$$

Frequency is in GHz. The equation predicts about 30K at 1 GHz and 40K at 10GHz.

7 Finding A_e/T_{sys} Directly from an Extended Source

The goal is to determine the ratio A_e/T_{sys} from measured and *a priori* data, where $A_e = \eta A_p$, with A_p being the physical area, η the aperture efficiency and T_{sys} the system temperature. In this section we work out the factor by which the the observed antenna temperature must be multiplied for a resolved source like the moon to emulate an unresolved observation. The effective collecting area, A_e, follows directly from this.

7.1 Power Received from an Antenna

Starting with Equation (12), the power received at the antenna terminals may be written

$$P_{sky}(\theta_a, \phi_a) = \int_{4\pi}\int B(\theta, \phi) A_{eff}(\theta, \phi; \theta_a, \phi_a) d\Omega \equiv kT_A(\theta_a, \phi_a) \tag{23}$$

where the subscript "a" denotes the antenna boresight direction. Note that here $A_{eff}(\theta, \phi; \theta_a, \phi_a) = A_{eff}(|\theta - \theta_a|, |\phi - \phi_a|)$ is the effective area in a given pointing direction and $A_{eff}(\theta - \theta_a, \phi - \phi_a)$ includes all effects in receiving the signal related to the beamshape. Factor out the effective area at boresight A_e (i.e. the point source response, $A_e = A_{eff}(0, 0)$, which is the maximum value) and let the normalized pattern be

$$H(\theta - \theta_a, \phi - \phi_a) = A_{eff}(\theta - \theta_a, \phi - \phi_a)/A_e \tag{24}$$

so now $H(0, 0) = 1$ and

$$P_{sky}(\theta_a, \phi_a) = A_e \int_{4\pi}\int B(\theta, \phi) H(\theta - \theta_a, \phi - \phi_a) d\Omega \equiv kT_A(\theta_a, \phi_a) \tag{25}$$

Note also that (Kraus Eq. 6.1, page 6-2 and Eq. 6-15, page 6-5)

$$\int_{4\pi}\int H(\theta,\phi)d\Omega \equiv \Omega_A = \lambda^2/A_e \tag{26}$$

where Ω_A is the beam solid angle in sr.

Define several other temperatures (notations not meant to be consistent with Ulich and Kutner):

T_{A*} = contribution due to integrating over the source

$T_{A\prime}$ = contribution due to integrating over the rest of the sky (becomes part of T_{sys}) therefore

$$P_{sky} = kT_{A\prime} + kT_{A*} \tag{27}$$

Let the flux density per beam of a source be

$$S_b \equiv \frac{2kT_A*}{A_e} = 2\int_{src}\int B(\theta,\phi)H(\theta-\theta_a,\phi-\phi_a)d\Omega \tag{28}$$

and we see that

$$P_{sky}(\theta_a,\phi_a) = kT_{A\prime} + \frac{1}{2}A_eS_b \tag{29}$$

The factor of two comes about because we normalize the flux density to both polarizations and assume an unpolarized source. The quoted or computed flux density value is

$$S = 2\int_{src}\int B(\theta,\phi)d\Omega \tag{30}$$

which, for a circular disc of diameter θ_s at uniform temperature T_s, may be expressed as

$$S = 2\int_0^{\theta_s/2}\int_0^{2\pi}\frac{kT_s}{\lambda^2}\sin\theta d\phi d\theta = \frac{4\pi kT_s}{\lambda^2}[1-\cos(\theta_s/2)] = \frac{2kT_s\Omega_s}{\lambda^2} \tag{31}$$

where Ω_s is the disc size in sr.

7.2 Results Assuming a Gaussian Beam

So far, we have made no assumptions. To proceed, however, we need to estimate the beam flux density, S_b, given the flux density, S. To do so one must know or assume the beamshape and it is convenient to assume the beam pointing is centered on the source ($\theta_a = 0$) and it is axially symmetric. A Gaussian beam matched at the FWHM is often a good approximation:

$$H(\theta) = e^{-4\ln 2(\theta/\theta_{3dB})^2} \tag{32}$$

Here θ_{3dB} is the FWHM so the half-power point is at $\theta = \theta_{3dB}/2$. Recall the two conditions, namely that $H(0,0) = 1$, which is clearly satisfied and

$$\int_{4\pi}\int H(\theta,\phi)d\Omega \equiv \Omega_A = \lambda^2/A_e \tag{33}$$

Proceeding to evaluate this yields:

$$\int_{4\pi}\int H(\theta,\phi)d\Omega = \int_0^{\pi}\int_0^{2\pi} e^{-4\ln 2(\theta/\theta_{3dB})^2}\sin\theta d\phi d\theta = 2\pi\int_0^{\pi} e^{-4\ln 2(\theta/\theta_{3dB})^2}\sin\theta d\theta \tag{34}$$

Note that if $\theta_{3dB} \ll \pi$, then

$$2\pi\int_0^{\pi} e^{-4\ln 2(\theta/\theta_{3dB})^2}\sin\theta d\theta \approx 2\pi\int_0^{\pi} e^{-4\ln 2(\theta/\theta_{3dB})^2}\theta d\theta \tag{35}$$

which may be evaluated to yield

$$\int_{4\pi}\int H(\theta,\phi)d\Omega \approx \frac{\pi\theta_{3dB}^2}{4\ln 2} \tag{36}$$

For an ATA antenna,

$$\theta_{3dB} \approx 1.24\frac{\lambda}{D} \tag{37}$$

so therefore

$$\int_{4\pi}\int H(\theta,\phi)d\Omega = \frac{\lambda^2}{\frac{16\ln 2}{(1.24\pi)^2}A_p} = \frac{\lambda^2}{A_e} \tag{38}$$

implying an efficiency of 73% for this illumination pattern. Obviously the real efficiency is less than this, however, if the beamshape is well-approximated by the Gaussian across the source of interest, the computation of the flux density per beam should be accurate. That is, we can write the beam pattern as

$$H(\theta) = f(\theta)e^{-2\ln 2(\theta/\theta_{3dB})^2} \tag{39}$$

where $f(\theta < \theta_{3dB}/2 \approx 1$ and defined such that the integral evaluates correctly. This then yields the integral

$$S_b = 2\int_0^{\theta_s/2}\int_0 2\pi\frac{kT_s}{\lambda^2}f(\theta)e^{-2\ln 2(\theta/\theta_{3dB})^2}\sin\theta d\phi d\theta \tag{40}$$

If we assume that the source is small, such that $\sin\theta \approx \theta$ and $f(\theta) \approx 1$, then the integration may be performed to yield

$$S_b = \left(\frac{2kT_s}{\lambda^2}\right)\left(\frac{\pi\theta_{3dB}^2}{4\ln 2}\right)\left(1 - e^{-\ln 2(\theta_s/\theta_{3dB})^2}\right) \tag{41}$$

Checking the expression at the various limiting cases is useful.

7.2.1 Point Source

In the case of a point source $\theta_s \ll \theta_{3dB}$ then $S_b \approx (2kT_s/\lambda^2)\pi\theta_s^2/4 \approx S$ as expected.

7.2.2 Enclosed Antenna

In the case where the antenna is in a black box at temperature T_s, it is useful to start at Equation 42

$$S_b \equiv \frac{2kT_{A*}}{A_e} = 2\int_{src}\int B(\theta,\phi)H(\theta-\theta_a,\phi-\phi_a)d\Omega \tag{42}$$

which then yields

$$\frac{2kT_{A*}}{A_e} = 2\int_{src}\int\frac{kT_s}{\lambda^2}H(\theta,\phi)d\Omega = \frac{2kT_s}{\lambda^2}\int_{src}\int H(\theta,\phi)d\Omega = \frac{2kT_s}{\lambda^2}\Omega_A = \frac{2kT_s}{\lambda^2}\frac{\lambda^2}{A_e} \tag{43}$$

or

$$T_{A\prime} = T_s \tag{44}$$

as expected (making use of Eq. 26).

7.3 Source Beam Correction Factor

Define the source beam correction factor, ϵ, such that $S_b = \epsilon S$. In the case of the Gaussian beam of the previous section, this yields

$$\epsilon = \frac{S_b}{S} = \frac{\pi\theta_{3dB}^2}{4\ln 2\Omega_s}\left(1 - e^{-\ln 2(\theta_s/\theta_{3dB})^2}\right) \tag{45}$$

Approximating $\Omega_s \approx \pi\theta_s^2/4$ and letting $\zeta = \ln 2(\theta_s/\theta_{3dB})^2$ yields

$$\epsilon = \frac{1 - e^{-\zeta}}{\zeta} \tag{46}$$

7.4 Y-Factor Test

Let $P_{sky}(0,0) \equiv P_{on}$ and $P_{sky}(\theta_a \gg BW, \phi_a) \equiv P_{off}$ then the "Y" measurement may be expressed as

$$Y = \frac{P_{on}}{P_{off}} = \frac{kT_{sys\prime} + (kT_{A\prime})_{on} + (kT_{A*})_{on}}{kT_{sys\prime} + (kT_A)_{off}} \approx 1 + \frac{T_{A*}}{T_{sys}} \tag{47}$$

where we've assumed that $(T_{A*})_{off}$ is negligible (i.e., that the power contributed by the source in the sidelobes is negligible) and that the power in the different pointing directions is the same (typically good for constant elevation, if RFI can be discerned and if the measurements are taken fairly rapidly relative to the gain stability). Therefore

$$Y = 1 + \frac{\frac{1}{2}S_b A_e}{kT_{sys}} \tag{48}$$

and solving for the the quantity of interest yields

$$\frac{A_e}{T_{sys}} = \frac{2k}{S_b}(Y-1) \tag{49}$$

and finally

$$\frac{A_e}{T_{sys}} = \frac{2k}{\epsilon S}(Y-1) \tag{50}$$

The reference metric is relative to A_p/T_{sys}, that is we are comparing to a perfect, uniformly illuminated antenna. We can therefore state an area normalized metric of

$$\frac{\eta}{T_{sys}} = \frac{2k}{\epsilon S A_p}(Y-1) \tag{51}$$

with our milestone value being 0.92%/K.

7.5 Antenna Temperature

It is interesting to calculate the expected antenna temperature from our canonical disc-like source at temperature T_s. Recall

$$T_{A*} = \frac{A_e S_b}{2K} \tag{52}$$

where S_b is given by Equation 41. Reducing this and using Equation 37 yields

$$T_{A*} = \eta \frac{(1.24\pi)^2}{16 \ln 2} T_s \left(1 - e^{-\ln 2(\theta_s/\theta_{3dB})^2}\right) = \frac{\eta}{0.731} T_s \left(1 - e^{-\ln 2(\theta_s/\theta_{3dB})^2}\right) \tag{53}$$

It is interesting to note that the wavelength dependence of the flux cancels with the wavelength dependence of the square of the beamwidth. Assuming an antenna efficiency of $\eta = 58\%$ at 7 GHz (where $\theta_s = \theta_{3dB}$ for the moon or sun) yields $T_{A*} = 0.397 T_s$.

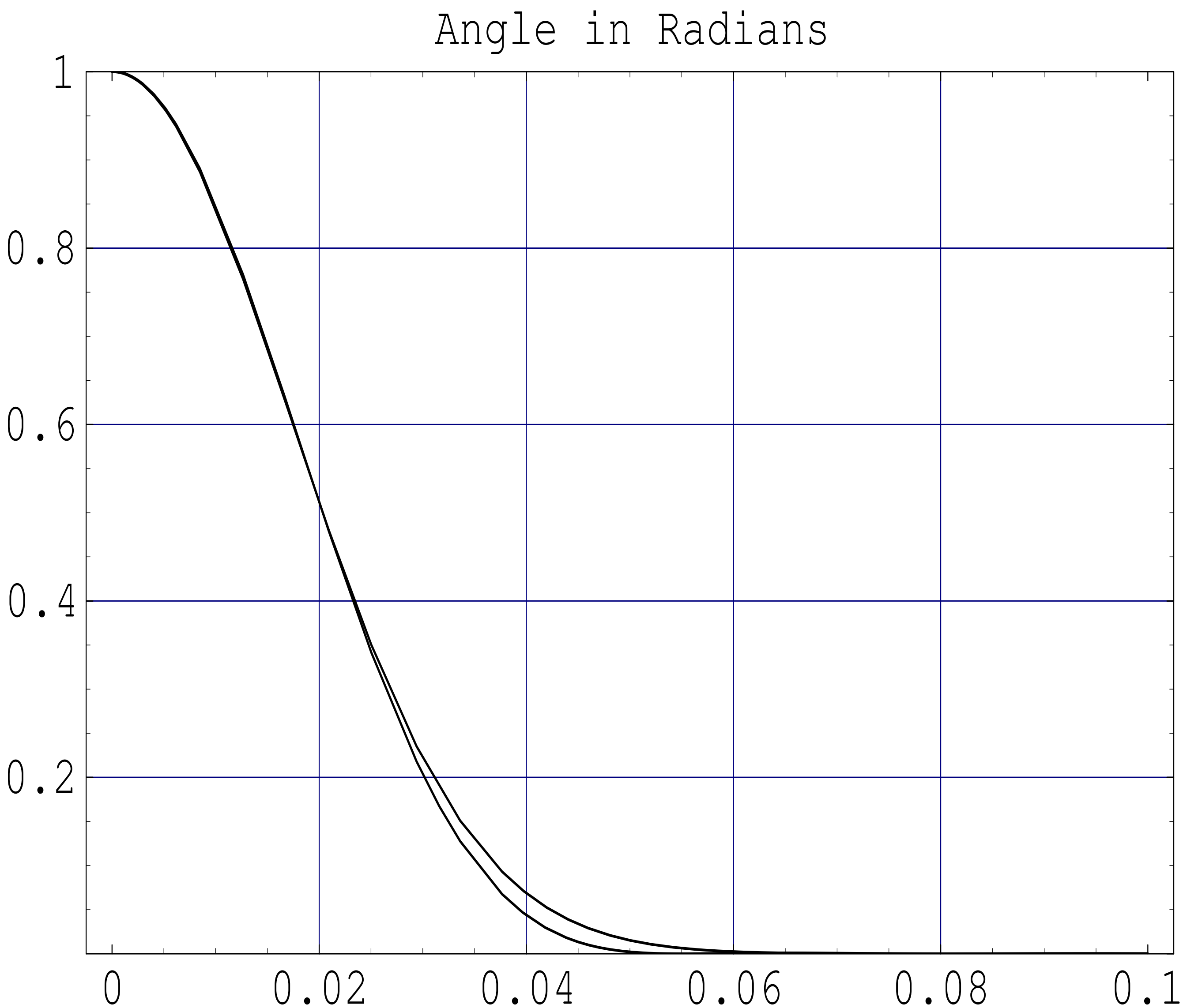

Figure 1: ATA antenna gain functions at angles close to bore sight. The bottom curve is the theoretical gain function calculated from the diffraction pattern. The upper curve is a Gaussian function chosen to have the same FWHM as the lower curve. The Gaussian curve integrated over $0 \leq \phi \leq 2\pi$ and $0 \leq \theta \leq 40^o$ contains 7% more area than the the lower curve.

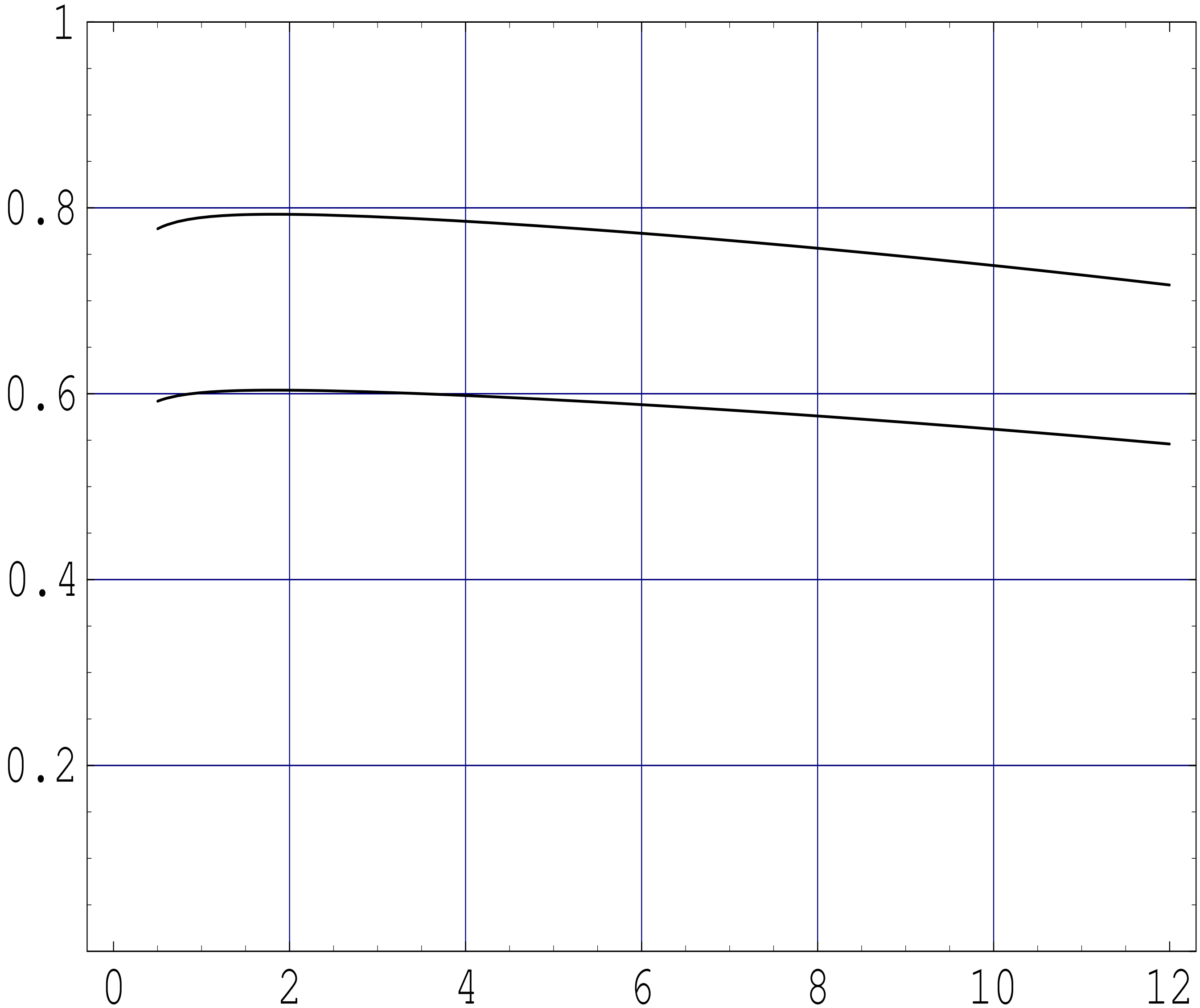

Figure 2: Plots of the expected beam efficiency and aperture efficiency as functions of frequency. The abscissa is frequency in GHz. The upper is the beam efficiency, to be used with a Gaussian beam whose FWHM is $2.33^o(\lambda/.20m)$. The lower is the aperture efficiency, a fraction of the antenna geometric area of $29.22\ m^2$.

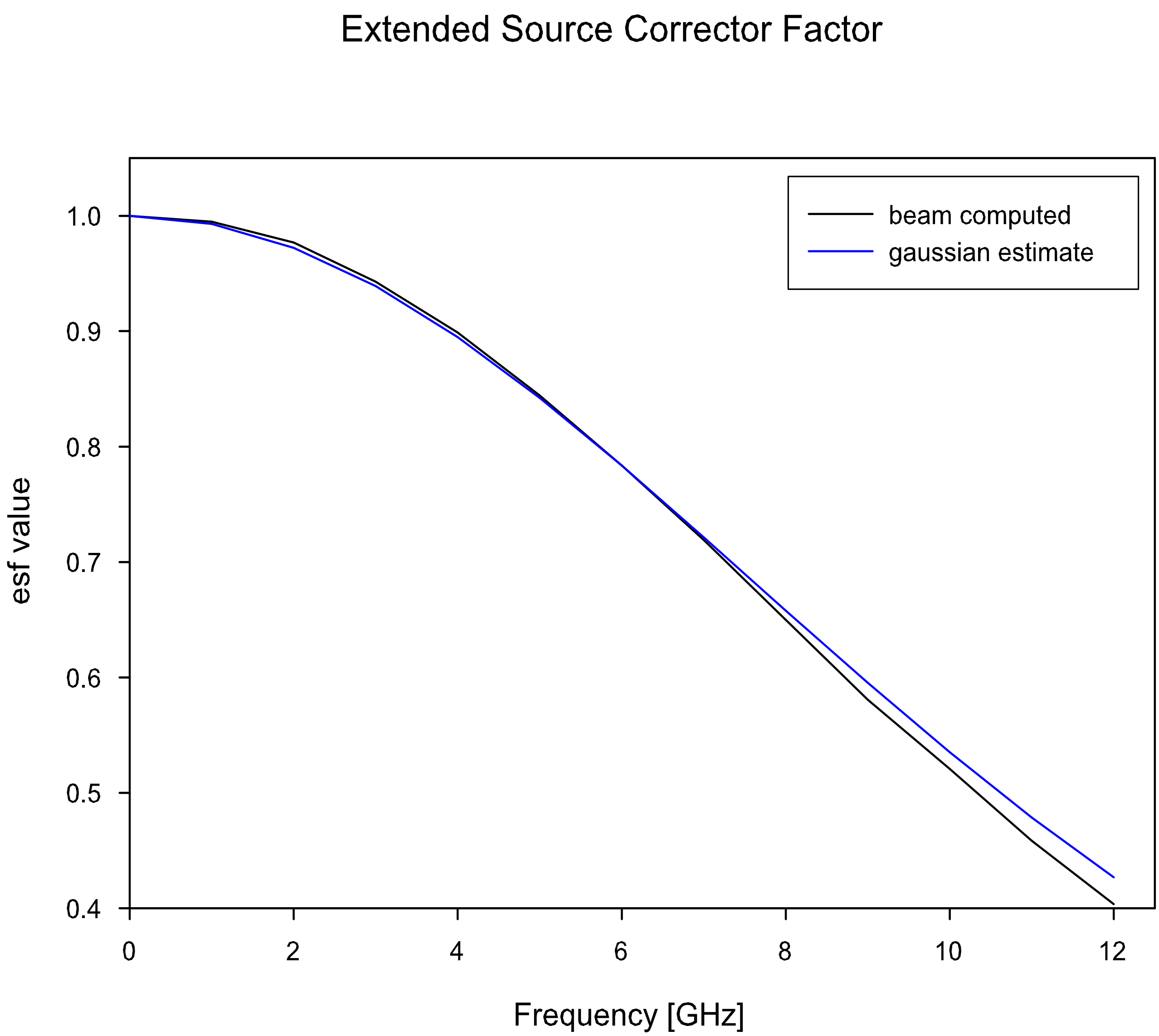

Figure 3: The extended source correction factor for the moon for the ATA as a function of operating frequency. The upper line at high frequencies corresponds to the Gaussian approximation to the beam, and the lower curve is based on the actual theoretical beam.