The "SETI Efficiency" of Array Radio Telescopes

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Reasonable estimates of the number of detectable civilizations in the Milky Way suggest that it will likely be necessary to examine perhaps ten million stars to have a substantial probability of having chanced upon a radio-transmitting civilization. With or without success after examining so many stars, it will be desirable to examine even more, eventually examining with high sensitivity most of the stars of the Milky Way. In order to examine so many stars in an appealing time, SETI radio telescopes need to be designed so as to examine the greatest volume of space, thus the maximum number of stars, at any given time. Even in the case where the basic search strategy is that of a targeted search, there is still a benefit to searching as large a volume of space as possible in addition to the target star.

The volume of space examined by a telescope, given a reference set of transmitter and receiver parameters, such as transmitter EIRP, system noise, channel bandwidth, and integration time, is proportional to

$$G^{3/2}\Omega \qquad (1)$$

where G is the maximum gain of the array, and Ω is the solid angle of the main beam. (1) above shows that a SETI telescope should be built to have, simultaneously, maximum gain and maximum beam size. These are conflicting goals. From the Second Law of Thermodynamics, the maximum Ω is limited to $< 4\pi/G$, and is maximum when the array is as compact as possible. Indeed, the maximum possible Ω (with full gain), from the Calculus of Variations is that of a filled circular aperture with the same collecting area as that of the array.. Unfortunately, then, all arrays will be less effective as SETI instruments than a filled aperture of the same collecting area, a price which is paid for using an array approach to achieving large collecting area at low cost.

In view of the fact that the maximum G of the array is independent of its configuration, and is equal to the gain of the equivalent filled aperture, the volume of space examined by the array relative to the maximum possible is just proportional to (. This suggests that we can define a "SETI Efficiency", "SE" ,of an array as simply

$$SE = \Omega_{array} / \Omega_{filled\ aperture}$$

Here, of course, $\Omega_{filled\ aperture}$ represents the Ω of the circular filled aperture with the same collecting area.

The SE is just a quantitative measure of the volume of space being examined in practice by the array, compared to the maximum possible with its collecting area.

The SE obviously is a very valuable, indeed essential, criterion to use in evaluating the efficacy of array configurations.

Note that this efficiency is as important a measure of quality for any system intended to search for new objects as it is with SETI. A good example is a system to search for new pulsars. Indeed, the ôSEö might be called, more generally, the ôSearch Efficiencyö.

Applications of the SE concept:

1) A circular array of parallel rows and columns of circular dishes, all just touching one another. In this example, and with the calculations for all the array configurations to follow, it is assumed that the illumination pattern (the ôweightingö) for the filled aperture or array, is the same. Only the extent of the array north-south and east-west is varied. At the zenith, the SE of this array is just the ratio of the area of a circle to the area of a square of the same dimension. Thus:

$$SE = \pi/4 \text{ or } \underline{0.79.}$$

Note that vignetting occurs at all zenith angles other than zero.

2) The most compact array possible with circular dishes, which is a honeycomb, or hexagonal configuration This is the same basic geometry as in the Cyclops Report. The SE is improved by the secant of 30 degrees in this case. Thus the zenith SE is

$$SE = \underline{0.91.}$$

Note that best SE is obtained with a constant dish spacing. Any variation of the dish spacing either north-south or east-west in this, and the examples to follow, will decrease the SE. Either vignetting will set in for some dishes sooner than others where the dish spacing is less than the nominal, or the beam will be broadened by spacings more than the nominal. In view of this, greatest SE may lead to grating sidelobes. However, these are of little consequence to SETI. It may be possible to reduce such lobes by dithering the dish positions at the cost of an insignificant loss in SE.

This is the maximum SE which can be achieved with circular dishes. If one had available rectangular or hexagonal dishes which could be fed with the same aperture efficiency as a circular dish, it would be possible to arrange them so that the projected aperture was filled, but for just two pointing directions. This example, only of academic interest, would achieve SE =1.0.

Both of these configurations are very undesirable, however, since any steering of the dishes away from zenith leads to vignetting ("shadowing"). Furthermore, when shadowing is occurring, the pickup of ground radiation will likely increase substantially, increasing system noise. Some compromise between high SE and avoidance of vignetting must be made,

3) Arrays designed to avoid vignetting up to an arbitrary zenith angle Z. The separation of antennas required to avoid vignetting up to a zenith angle of Z is given simply by sec Z. Here are some examples:

a) A circular honeycomb array in which the dishes can be steered 20 degrees from the zenith without vignetting. This provides for unvignetted tracking for as much as almost three hours. Based on experiences at Arecibo, this is a very useful, thus interesting arrangement. Note that, in contrast to the Arecibo situation, observations can in fact be made at all zenith angles, but with decreasing gain at higher zenith angles. The required dish separation is just 1.06 dish diameters. The SE in this case at zenith is

SE = 0.81.

Note that this provides for 40 degrees of declination coverage without vignetting. The loss in SE is not enormous for this desirable arrangement. Note the fact that the SE actually increases as one goes away from the zenith, reaching a maximum at the largest unvignetted zenith angle.

a) An array of similar geometry but spaced so as to give 30 degrees of unvignetted zenith angle coverage. The required dish separation is now 1.15 dish diameters. In this case, the SE at zenith is

SE = 0.68.

This provides 60 degrees of unvignetted declination coverage.

Note that the zenith SE declines rapidly and nonlinearly with greater unvignetted zenith angles.

a) The historic example of the proposed Cyclops array. Here, the Cyclops Report argued that a compact array was desirable to minimize construction costs. It had in mind targeted searches, and did not consider the advantage of a compact array for high spatial volume searches. Nevertheless, the report called for an unvignetted zenith angle of 70 degrees to provide large invignetted sky coverage. This zenith angle requires dish spacings of 3 dish diameters both north-south and east-west. This would have led to a SE of about

SE = 0.10

This would have been a very poor design for a SETI search system. It would have lengthened search times by a large factor for comprehensive searches of the sky.

The above results suggest that, in practice, it is probably desirable to have greater dish spacings north-south than east-west in order to provide substantial declination coverage while keeping the SE high. The east-west spacing mostly limits the unvignetted tracking time, whereas the north-south spacing controls SE as a function of declination. Large declination coverage is very desirable for SETI, arguing for the larger north-south spacing. Obviously, studies of the SE of various configurations, as a function of hour angle and declination, should be made. These can inform choices of the most desirable compromise configurations.

For all SETI telescopes, it is very desirable that the telescope be able to observe the region of the galactic center with high efficiency. The galactic center is at high zenith angle Z for sites in the U.S. Indeed, this consideration greatly favors sites in the Southern Hemisphere. Here are some examples:

1) The ideal SETI telescope. Since the galactic plane extends from û62 to +62 degrees declination, the optimum SETI telescope should observe this range of declinations with high efficiency. This calls for the telescope to be placed on the equator, with unvignetted coverage to at least Z = 62 degrees in the north-south direction. This

requires a dish separation of 2.1 dish diameters. If 20 degrees of unvignetted hour angle motion is provided, the SE at the declination extremes is 0.81, as before, and at the zenith, declination 0, the SE is

SE = 0.39.

Thus the ideal SETI telescope would operate at about half the maximum possible efficiency, but the payoff would be good coverage of all of the disk of the galaxy.

1) The ATA: If the ATA, situated at latitude +41 degrees, is to observe the galactic center at û30 degrees with maximum efficiency, then the north-south spacing of the antennas needs to be that for Z= 71 degrees. This requires a north-south antenna spacing of 3.07 antenna diameters. With zero east-west spacing this produces the minimum SE, as usual at the zenith, of

SE = 0.30.

If 20 degrees of hour angle coverage is allowed, the minimum SE becomes

SE = 0.27.

This combination of Z coverage and telescope latitude allows the telescope to observe far less of the galaxy than the telescope located on the equator, as well as having a substantially lower SE in general. It does give a SE about 3 times better than the canonical Cyclops SE, which is a substantial benefit.

Note that the diameter of a circular filled aperture of 1 hectare is 113 meters.
The dimensions of an elliptical ATA having the parameters above would be about 390 meters north-south by 120 meters east-west.

A note about sidelobe levels. The second law of thermodynamics requires, for an antenna system which is linear with square law detectors, that

(Gd(over 4(steradians = 4(

Then

(Gd(over the main beam + (Gd(over the sidelobes = 4(or,

to a good approximation,

G_{max}(+ 4($G_{sidelobes}$ (4(

where $G_{sidelobes}$ = the mean sidelobe level.

This means that a larger beam solid angle (leads to reduced sidelobe levels. This is easily understood to result from the fact that larger (means a more compact array, and thus fewer missing spacings in the u-v plane. Thus lower sidelobe levels are an added benefit accruing to increased SE.

Some General Rules of Thumb: From the above discussion, there emerge several useful general rules of thumb:

a) The SETI Efficiency is always less than a single filled aperture with the same collecting area.

a) The maximum SETI Efficiency for realistic arrays is 0.91.

a) Lesser unvignetted tracking times improve SE.

a) The maximum volume of space searched by an array telescope is proportional to the square root of the collecting area, no matter what the arrangement of the collecting area.

a) More compact arrays are always desirable.

a) The minimum SETI efficiency, except when there is vignetting, occurs at the zenith.

a) The maximum SETI efficiency occurs at the zenith angle at which vignetting sets in.

a) Highest SETI efficiency occurs when antennas are spaced equally, one from the other. This enhances crating lobes, but these are of very minor consequence to SETI, and may be reduced by dithering the dish positions.

a) A honeycomb distribution is always advantageous.

a) The north-south spacing of antennas should be derived from the choice of zenith angle at which highest efficiency is desired.

a) The east-west spacing of antennas is determined by a largely subjective choice of desired unvignetted hour angle coverage, or unvignetted tracking time.

a) The projection of the array on the sky should be elliptical.

a) Higher SETI Efficiency leads to lower average sidelobe levels.

a) The considerations here apply equally well to systems designed for other searches, such as for pulsars, quasars, etc. Indeed, the SE might be called, more generally, the ôSearch Efficiencyö

Subjects which should be explored in the future include at least a) the calculation of SE as a function of zenith angle, including zenith angles where vignetting is occurring; and b) the mean SE for various array configurations averaged over all available zenith angle and hour angle combinations.

Comment:

Date: Tue, 15 Aug 2000 16:21:41 +1000
From: Ron Ekers <rekers@atnf.csiro.au>
To: drake@seti.org, 1hT@seti.org
Subject: SETI Efficiency

Frank,

Your analysis omits the effect of the multiple phased array beams which can be formed to fill the primary beam of the elements of the array. Given the relative cost of computing and antenna hardware and rate of change of these costs I have always assumed that you would generate the maximum possible number of beams unless you were making a targeted search.

If you include this I think you will find that all arrays outperform a single dish by a large factor. That the larger arrays will require more phased array beams to fill the primary beam, so are in that sense less efficient in that they take more hardware to achieve the same efficiency.

If you put a focal plane array in the single dish to generate more beams and hence greater search efficiency you will find a rather simple and magic formulae which shows that the single dish is equivalent to an array of the same total area when the number of receivers in the focal plane is equal to the number of elements in the array. For large n arrays this becomes impossible in a single dish because of the optics unless you switch to something like a Luneburg lens.

Ron

Reply:

Date: Tue, 15 Aug 2000 09:44:47 -0700
From: Frank Drake <drake@seti.org>
To: Ron Ekers <rekers@atnf.csiro.au>
Cc: 1hT@seti.org
Subject: Re: SETI Efficiency

Hi Ron:

I agree completely with what you say. It is, of course, possible to synthesize beams to fill the primary beam of the array elements. However, the larger the synthesized beams, the fewer are needed to fill the element primary beam. Each beam needs a spectrum analyzer, and with the ambitious bandwidths being proposed for the ATA these are very expensive, require expensive fiber optics links, etc. More data analysis capability is required, too. Arrays which synthesize smaller beams are more extended and thus more expensive in real estate and signal connections. So there is a substantial advantage, I believe, in synthesizing large beams with the array. There is no downside to it if one's goal is a good search instrument. Better to put your money into more collecting area.

Frank