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Array Processing Estimates for the ATA

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Summary: Array processing estimates XF vs FX

XF

The dominant computing for XF correlation is the complex multiply accumulations (CMAC) at the bandwidth rate (assuming complex samples). For N antennas in the array the required processing rate is

$$\left[\frac{N(N-1)}{2}\right]BL \qquad \text{CMACs/second}$$

for a bandwidth B and L lags. If some spectral resolution is required then more than 1 lag is required. The spectral resolution of XF correlator has improved spectral resolution by dividing the bandwidth B into M sub-bands. In this case the required processing rate remains unchanged.

FX

The FX correlator is one in which the division of the total bandwidth into sub-bands is accomplished by a Fourier transform. In this case the processing rate required by FFT is

$$N\,B\,\log_2 n \qquad \text{CMACs/second}$$

where n is the number of spectral points or the length of the FFT. If we require “direct” uv data for each baseline then we need to add

$$\frac{N(N-1)}{2}B \qquad \text{CMACs/second}$$

for the “X” part of the FX correlator. However, it is more efficient to derive a direct image from each frequency channel of the FFT output. If this is acceptable then the gain is enormous for large arrays since we can avoid baseline processing. For a square array on a regular grid the “snapshot” image calculations require

$$N\,B\,\log_2 N \qquad \text{CMACs/second}$$

In practice, a snapshot from an array with irregular spacing would require a value of N larger than the number of antennas but $N\log_2 N$ is still likely to be much smaller than the number of baselines.

Direct image accumulation

An efficient processing method for large arrays is as follows:

1] Sample and Fast Fourier transform the data from each polarization from each antenna. In practice this might be done by first converting the R.F. signals into 50 MHz chunks using analog frequency conversion. Then the 50 MHz chunks would be sampled and converted to 12 bit digital signals. These 50 MHz bandwidth (100 MHz clock) might be further divided into 5 MHz I/Q chunks using a digital baseband converter (like the GrayChip 4016) for the FFT processing with a DSP (like the G4).

$$x_k(t) \xrightarrow{\text{FFT}} x_k(\omega)$$

where t = time
k = antenna number
ω = frequency

The size of the FFT would be chosen so that n/bw >> array dimension

where n = number of complex samples in FFT
bw = bandwidth of I/Q chunk

For example if n=1024 and bw = 5 MHz one FFT corresponds to a 204.8 microsecond block of data with 5 KHz resolution.

2] Form beams at each frequency by summing the complex FFT outputs from each antenna for a single polarization.

$$b_s(\omega,\theta) = \sum_k x_k(\omega) e^{-i\omega\tau_k(\theta)}$$

where $\tau_k(\theta)$ is the signal delay relative to a fixed reference point
θ = angle in the sky
s = time segment number

If the array is uniform all the beams can be formed by a 2-D FFT. If only a few beams are needed a DFT is adequate.

Beams for a range of frequencies can be transformed to beams for a range of time samples within a time segment

$$b_s(t,\theta) = \int_{-\pi B}^{\pi B} b_s(\omega,\theta) e^{i\omega t}\, d\omega$$

for better time resolution needed for transient phenomena and pulsars.

3] Form power beams by taking the magnitude squared of each beam

$$|b_s(\omega,\theta)|^2$$

which can then be accumulated over as many segments and frequencies as desired to form an accumulated image

$$I(\theta) = \sum_s \sum_\omega |bs(\omega,\theta)|^2$$

It can be shown that image averaging is similar to uv averaging. For a single frequency bin

$$I(\theta) = \sum_s \left| \sum_k x_k e^{-i\omega\tau_k(\theta)} \right|^2 = \sum_s \sum_k x_k e^{-i\omega\tau_k} \sum_\ell x_\ell^* e^{i\omega\tau_\ell}$$
$$= \sum_s \sum_k |x_k|^2 + \sum_s \sum_{k\neq\ell} x_k x_\ell^* e^{-i\omega(\tau_k-\tau_\ell)}$$

If it is desirable to use the standard radio astronomical imaging and cleaning methods the images can be averaged for a duration for which the uv changes are small and inversed transformed to the uv plane. This operation might be performed every few minutes which would allow the use of a DFT to calculate the correlation for the uv points appropriate for the actual antenna locations in the array. For a non-uniform array the image $I(\theta)$ is the "dirty" image whose inverse transform has zeroes at uv points from which there are no corresponding baselines. A 2-D FFT can be used for the forward transform to obtain the voltage beam and reverse transform to the uv plane for a non-uniform array by using a large enough FFT to satisfy the Nyquist sampling theorem. The antenna based signals can then be placed into the 2-D array using a 2-D "multirate" interpolation filter.

<u>Approximate estimate of IF and DSPs for ATA</u>

Assume a) 500 dual polarized antennas
b) 50 MHz BW expandable in chunks of 50 MHz

Number analog IF conversion modules	1000
Number A/D converters	1000
Number of 4016 Gray chips[1] (3 per A/D)	3000
Number of G4 DSPs to FFT output of GrayChips[2]	3000
Number of G4 DSPs for radio astronomy beamforming	2000

For 500 MHz bandwidth all but the IF conversion are multiplied by a factor of 10.

[1] GrayChip 4016 provides 4 I/Q outputs with selectable IF frequency and up to 5 MHz wide channels
[2] G4 DSP does 1024 complex FFT in 30 microseconds or approx. 300×10^6 CMACs/sec