Forming Nulls toward Satellites

Wm. J. Welch

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Abstract

For an overall size of about 2km for the 1ht, it is practical to form nulls in the directions of satellites in order to reduce the amount of RFI from the satellites. The depth of a null that one may expect to achieve is of the order of -30 db. For a simple model of an array as a linear array with equally spaced and equally excited elements and overall extent of 2km, the full width at -30 db of the nulls is found to be $0.6''$ at a wavelength of 20cm. The beam width is about $10''$ for this array at this wavelength. Thus, locating a satellite in a null at a level of -30db or better means knowing the position of the satellite to better than 1/20 of a beamwidth for the array. In addition to knowing the satellite positions to this accuracy, it is necessary to know the antenna relative locations with sufficient accuracy to be able to predict the null positions to within 1/20 of a beamwidth. In order that the array achieve deep nulls, the RMS errors between the elements must be sufficiently small. For example, a -40db null requires that the RMS relative positon errors between the elements be 1/30 of a wavelength. This should be possible.

For the GPS satellites at an orbital radius of about 20,000 km, one half of the geosynchronous orbit, the RSS positional uncertainty of 5.8m with Selective Availability turned off corresponds to an angular uncertainty of about $0.1''$ on the ground, significantly smaller than the $0.6''$ null width above. It should be possible to null the emissions from lower LEO satellites, down to about 1200km above the earth surface, with the same positional uncertainty as available for the GPS satellites.

1 Introduction

Two general schemes are studied for the mitigation of RFI in radio telescope observations. In one, a measurement of the RFI signal is obtained at high signal/noise and it is then subtracted from the signal in the main beam. In the other, an array null is formed toward the unwanted interference. The RFI is then rejected according to the depth of the null. The advantage of the latter scheme is that RFI which enters at low signal/noise may be rejected at the level of the null, whereas the former scheme requires high interference/noise to be effective. The weak signals of the GPS satellites are examples of weak RFI. Only if the GPS signal is demodulated and can be measured (integrated) over a long time interval does it have sufficient signal/noise to be accurately subtracted from the desired signal.

It is therefore an important question whether the 1ht can place a null on an interfering satellite, such as a GPS satellite, to eliminate its interference, even though it is weak. There are really two questions. 1. Are the satellite positions accurately enough known so that they would fall within the 30db width of an array null with a perfectly calibrated array? 2. Can we position the nulls on the satellites accurately enough?

2 Character of the Nulls for a Linear Array

A linear, equally spaced, array of N elements is a suitable simple model for the 1ht for the study of the null width. If the array spacing is d, the angle measured from the broadside direction is θ, and the wavelength is λ, then the voltage pattern is

$$V(\theta) = \frac{sin[\frac{2\pi N d sin\theta}{\lambda}]}{N sin[\frac{2\pi d sin\theta}{\lambda}]} \tag{1}$$

The interesting part of the pattern is near $\theta \sim 0$, so that $sin\theta \approx \theta$. The first null occurs when

$$\frac{2\pi N d\theta_n}{\lambda} = \pi \tag{2}$$

If ϵ is the half 30db width of the null, then $V(\theta_n + \epsilon)$=.03. The denominator in (1) varies slowly with θ and can be evaluated with θ =θ_n. Then the denominator becomes N$sin(\pi/N) \approx \pi$. With this substitution,

$$\epsilon = -\frac{(.094)\lambda}{2\pi N d} \quad (3)$$

The full 30db width of the null is 2ϵ.

3 Required Array precision

What precision in the array construction is required to achieve nulls that are sufficiently deep? Beckman and Spizzichino ("The Scattering of Electromagnetic Waves from Rough Surfaces", Macmillan, 1963) give a formula for the reflection from a rough mirror which we can adapt to this question.

$$\psi(\theta)^2 = \psi_0(\theta)^2 e^{-g} + g\frac{\pi T^2}{\lambda^2} e^{-(\frac{\pi T\theta}{\lambda})^2} \quad (4)$$

$\psi(\theta)^2$ is the gain pattern. T is the roughness correlation length across the surface. $g = (4\pi\sigma/\lambda)^2$ is the MS height phase error, where σ is the RMS height error. The first term is the ideal diffraction pattern, reduced by the roughness factor e^{-g}. The second term is the radiation scattered by the roughness. For our application, we replace 2σ by σ, since we have no reflection. The correlation length is approximately the typical horizontal element spacing, d. At the nulls only the second term remains, and we can compare its amplitude to the gain peak.

Here are some typical numbers. T=d=7.5m. $\sigma/\lambda = 1/10$. Then g= $(2\pi\sigma/\lambda)^2$=0.40. λ=.20m. With Nd=2km, the first null is at θ_n=5×10^{-5}. Then $e^{-}(\frac{\pi T\theta_n}{\lambda})^2 \sim 1$ at that null. With these various numbers substituted, the factor in front of the second term is 1700. This is the scattered signal at the null and is to be compared with the central magnitude of the first term, $G_o N \times e^{-.4}$. G_o is the gain of one of the dishes, about 3700 at λ=0.2m, and N=500 This term is 1.24×10^6, and the scattered signal at the null is about -25db relative to the peak. With better precision, σ/λ=1/30, the scattered signal at the null is -40db.

4 Discussion

For the GPS satellites observed with the 1ht, $\lambda \sim$ 20cm and Nd ~2km. The full 30db null width is $0.6''$ for an error free array. The GPS satellites lie at an approximate orbital radius of 20,000 km. That puts them at about 13,600 km above the earth's surface, with its radius of 6400 km. The RSS positional uncertainty of the GPS satellites is now about 6m with the Selective Availability turned off. That is an angle of $0.1''$ as observed from the 1ht, substantially smaller than the half width of the 30 db null. Thus the accuracy in the knowledge of the GPS satellite positions corresponds to an angle that is significantly smaller than the 30 db null width of the 2km 1ht.

The beamwidth of the linear array of length 2 km used for the model of the 1ht is $\sim \frac{\lambda}{2Nd} = 10''$ at λ20cm. Thus, in order that the satellite fall inside the 30db null width, the array must be pointed to within 1/20 of its beamwidth. Experience shows that this accuracy is often realized with aperture synthesis arrays operating at cm wavelengths, but careful calibration is required. For smaller or larger arrays, the beamwidth will be correspondingly smaller or larger. The null width will also scale in the same fashion. The general conclusion is therefore that a pointing accuracy of 1/20 of a beamwidth or better for the array will be necessary to insure that the satellite can be positioned within the 30db null width of the array.

For satellites in lower orbits, a fixed linear uncertainty of 6m in their positions would bring their angular position uncertainty up to the 30db null width at an altitude of about 1200 km. Better angular accuracies may be possible if the satellite ephemerides are obtained with the 1ht, if that is necessary.

In order that the array be able to achieve a null that is adequately deep, it must be calibrated and stable to a positional accuracy of the elements given by the above discussion. For example, for a -40db null, the element position errors must be less than $\lambda/30$ RMS. At λ3cm, that is 1mm in antenna position or optical fiber length.

5 Conclusions

- The answer to the first question is that for arrays of no greater extant than a few km, operating at cm wavelengths, the satellite positional accuracies (assuming that GPS is typical) should be good enough that they can be located within the 30db null width of the array.

- The answer to the second question is that the narrowness of the 30db null width requires that the array be pointing to an accuracy of 1/20 of a beamwidth or better. This is achievable but requires careful calibration. Relative position errors between the elements of the order of 1/30 of a wavelength RMS are necessary to achieve a null depth of about -40 db. This can be achieved with careful calibration or self-calibration, but it must be reasonably stable.