Cross Correlating Interferometer Beams for Discrimination of Radio ETI Signals

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Abstract

We describe a novel method of ETI signal discrimination that cross correlates two synthetic interferometer beams pointed in different directions. Given a candidate signal found in one beam, we can estimate the amplitude and phase of this signal in a second beam pointed in another direction, on the assumption it is an ETI signal. This estimate can be compared to the measured cross correlation between beams. If the estimate and measurement don't agree, then the signal is identified as RFI.

Introduction

One style of observations in the search for extra terrestrial intelligence (SETI) uses a high resolution digital spectrometer applied to radio signals incident from a single direction on the sky (a beam). The signals sought are nearly pure sinusoids, because these signals propagate with minimal distortion through the interstellar medium. Unfortunately, radio signals of all kinds are generated by man made devices (known as radio frequency interference or RFI), and the major challenge in SETI observations is to discriminate between true ETI signals and RFI.[1]

Radio interferometers like the Allen Telescope Array (ATA) produce multiple phased array beams within the primary antenna field of view (FOV) at the same time. This offers the chance to perform SETI observations on multiple targets at the same time. Parallelization increases the SETI search speed, and since the observations are simultaneous it is possible to compare signals found in different beams to help discriminate ETI and RFI.

A previous study[2] of multi-beam RFI discrimination considered the likelihood that uncharacterized RFI would appear with measurable power in two or more beams. Because RFI enters in a side lobe of the array beam, it is likely to appear in two or more beams at the same time. On the other hand, ETI signals originate far from Earth and will appear strongly only in the beam pointed at their direction of origin. In a lengthy observation (a few minutes), it is extremely unlikely that some RFI will appear, “by accident,” in only one beam, so any such signal is a strong ETI candidate. For example, on a 5 minute track with two beams on the 350 element ATA, the chances are only 1/1000 that an RFI signal differs by more than a factor of two in power. This probability is reduced by another factor of 10^3 for each 5 minute integration, so if the signal is stable the ETI signals can be identified with high reliability.

In this paper we extend the above ideas by noting that a real ETI signal appearing in one beam will appear in all the other beams with a predictable amplitude and phase. We predict the outcome of cross correlation of one beam (voltage) with another and compare this estimate with the observed cross correlation of the measured beams. We obtain a highly effective discriminator for ETI and RFI. As compared to the previously described technique which compares beam powers, the present method decreases the "probability of false alarm" by a factor of $\sqrt{N}$, where N is the number of antennas (a factor of ~20 for a 350 element array).

This improved discrimination is important for two reasons: 1) it increases the speed of our ETI search, presumably leading to ETI detection in a shorter time, and 2) it increases the detection speed for transient or scintillating ETI signals. The latter is important as we extend the search to radio frequencies above 3 GHz where interstellar scintillation causes detected signals to fluctuate with time.[3,4] We need to identify ETI signals quickly before scintillation makes them disappear (or the transmitter is turned away from us).

Description of the Problem

The candidate signal is represented as a time series, $S(t)$, as is the receiver noise on the i[th] antenna $n_i(t)$. Assuming these are the only two contributions in the frequency range of interest, the phased array beam voltage for beam *j*, V_j, has the form

$$V_j(t) = \sum_{i=0}^{N} g_i \left[p_i\, S(t) + n_i(t) \right], \qquad [1]$$

where N is the number of antennas and p_i represents the gain of the dish in the direction of the source (direction dependence suppressed). The factor

$$g_i = \exp\left(i\,(\vec{k}_B - \vec{k}_S) \cdot \vec{r}_i \right)$$

arises from the difference between the beam look direction ($\hat{k}_B$) and the direction to the source ($\hat{k}_S$), with wave number $k = \frac{2\pi}{\lambda}$, and $\vec{r}_i$ is the antenna position.

Let's call the beam where $S(t)$ appears most strongly the candidate beam, *C*. If the signal is really coming from the target direction, then $p_{C,i} \approx 1$. $\vec{k}_B = \vec{k}_S$ implies $g_{C,i} \approx 1$. In all other (off) beams *O*, $p_i \approx 1$ but $\vec{k}_B \neq \vec{k}_S$, so

$$g_{O,i} = \exp\left[i\,\varphi_{O,i} \right] \qquad [2]$$

where $\varphi_{O,i}$ is well approximated by a random variable[†].

[†] True because the different beams are arbitrarily pointed and the antenna positions on the ground are approximately random at the ATA.

If the signal is RFI,[‡]

$$g_i = |g_i| \exp\left[i\,\varphi_{O,i} \right] \qquad [3]$$

for *all* the beams, and $\varphi_{O,i}$ is still a random variable. In case [2], $\sum_{i=0}^{N} g_i$ is precisely the form of a 2D random walk with equal length steps. The expectation value of the length of this vector is $\sqrt{N}$.[5] In case [3], we have a 2D random walk with unequal length steps. No general, analytic solution for such case exists,[6] but various authors indicate that if the average step size is $\langle g \rangle$, the expectation value of $\left| \sum_{i=0}^{N} g_i \right|$ is close to $\sqrt{N} \langle g \rangle$.

Analysis for ET signal in candidate beam

For an ET signal, we insert [2] into [1] revealing

$$V_C \approx N\,S(t) + \sum_{i=0}^{N} n_i(t) \qquad [4]$$

and

$$V_O \approx \sqrt{N} e^{i\gamma}\, S(t) + \sum_{i=0}^{N} g_i\, n_i(t)\,. \qquad [5]$$

Here we have used $\sqrt{N} e^{i\gamma} = \left\langle \sum_{i=0}^{N} g_i \right\rangle$, taken from the 2-D random walk results mentioned above. The factor γ is unknown and varies slowly with time depending on the synthetic beam pattern (0.001 Hz typical at ATA). We'll suppose it is constant over the measurement.

Suppose we have M samples of each of these voltages. The detected power in candidate or off beam, respectively, is

$$P_{CC} \equiv \sum_{m=0}^{M} \left[V_{C,m}\, V_{C,m}^{*} \right] \approx N\,M \left[N \left\langle S^2 \right\rangle + \left\langle n^2 \right\rangle \right], \qquad [6]$$

and

$$P_{OO} \equiv \sum_{m=0}^{M} \left[V_{O,m}\, V_{O,m}^{*} \right] \approx N\,M \left[\left\langle S^2 \right\rangle + \left\langle n^2 \right\rangle \right] \qquad [7]$$

If we cross-correlate the candidate beam with one of the off beams, we obtain

[‡] With rare exception RFI originates in a far out sidelobe of the primary beam, because you don't point at known interferers. The primary beam side lobes are polarized and generally uncharacterized.

$$P_{CO} \equiv \sum_{m=0}^{M} \left[V_{C,m} \, V_{O,m}^{*} \right] \approx N\,M \left[\sqrt{N} e^{i\gamma} \left\langle S^2 \right\rangle + \left\langle n^2 \right\rangle \right] \qquad [8]$$

All detected powers have similar noise contributions, but the signal is strongest in the candidate beam, moderate strength in the cross correlation, and weakest in the off beam (that is, $P_{CC} \sim \sqrt{N}\, P_{CO} \sim N\, P_{OO}$). With regard to the candidate beam, the cross correlation is a better comparator than the off beam since the signal appears more strongly and we can better estimate its value.

Analysis for RFI signal in candidate beam

The analysis is the same, except that we use [3] instead of [2], and the signal is not preferentially found in any beam. We obtain

$$P_{CC} \approx P_{CO} \approx P_{OO} \approx N\,M \left[\left\langle S^2 \right\rangle + \left\langle n^2 \right\rangle \right]. \qquad [9]$$

Discussion

For 350 antennas, $\sqrt{N} = 19$. Therefore, if $\left\langle S^2 \right\rangle > 19 \left\langle n^2 \right\rangle$, we can usually measure an ETI signal even in the cross correlation P_{CO}. If we don't cross correlate but compare to P_{OO}, then we require $\left\langle S^2 \right\rangle > 350 \left\langle n^2 \right\rangle$ to see the signal in the comparator power.

Moreover, we can calculate $\sum_{i=0}^{N} g_i$ to high accuracy knowing the antenna positions and the positions of the candidate and off beams. We predict not only the magnitude of the cross correlation, but also its complex phase from

$$P_{CO} \approx \left(\sum_{i=0}^{N} g_i \right) \frac{P_{CC}}{N} + O\left((N+1) \left\langle n^2 \right\rangle \right). \qquad [10]$$

The comparison of prediction [10] and measurement [8] gives us a powerful tool to identify ETI signals. We may set a threshold for agreement in terms of the estimated signal to noise ratio, and thereby deduce the likelihood our candidate signal arrives from the target direction. By the same token, if we discover (as is usually the case) that the signal is not arriving from the target direction, we identify it as RFI.

Conclusion

In our previous work comparing beam powers (P_{CC} with P_{OO}),[2] the ability to discriminate RFI from ETI signals was limited by the probability that the RFI would "accidentally" not appear in the off beams. With the sensitivity improvement described here, the chances that RFI will be immeasurably small in the comparator detector is reduced by a factor of 20. This improvement may allow us to discover the first ET civilization 20 times faster than otherwise.

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[1] Ekers, R. D., Cullers, D. K., Billingham, J., & Scheiffer, L. K. ed. 2002, SETI 2020: A Roadmap for the Search for Extraterrestrial Intelligence, (Mountain View: SETI Press).
[2] Harp, G. R. (2005), Using multiple beams to identify radio frequency interference in the search for extraterrestrial intelligence, *Radio Sci.*, 40, RS5S18.
[3] Cordes, J.M. (1991), Properties of Interstellar Scintillations Relevant to SETI, Memo IWG/JMC.1.
[4] Cordes, J.M., Rickett, B.J. (1998), Diffractive Interstellar Scintillation Timescales and Velocities, Astrophysical Journal **507**, 846-860.
[5] http://mathworld.wolfram.com/RandomWalk2-Dimensional.html
[6] Barber, B. C. *The non-isotropic two dimensional random walk*, Waves in Random Media **3**, (1993) 243-256.