Some Thermal Issues Regarding the ATA Node Air System

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Abstract

This is an addition to the DAVCO report on the air treatment plans for the ATA nodes. A somewhat lower thermal conductivity for the soil is selected from sandy soil data found on the WEB. A calculation of the penetration of the thermal wave associated with the daily and annual periodic surface temperatures finds the 1/e depth in the summer to be about 11cm(.36 ft) for the daily wave. The depth of the annual wave is $\sqrt{365}$ times greater. The PVC pipe from the nodes to the antennas is buried at a depth of 2 ft, and at this depth the residual daily periodic temperature swing is reduced to 0.3% of the surface value. The annual variation is slightly reduced at this depth, leading to an average value of 70F for mid summer and 48F at mid winter at the depth of the PVC pipe, both based on surface temperature measurements. A model of the variation of the temperature of the air blown into the pipe at the node shows that there is an exponential decrease in the daily periodic temperature of the air temperature along the pipe. A thermal diffusion model for the periodic heat flow in the earth surrounding the pipe provides the run of thermal reactance of the earth as a function of the period. Entering values for the thermal resistances and heat convection, many taken from the DAVCO report, shows that for a pipe of length 50 feet, the swing in air temperature is reduced by the factor 0.29. For a 100 foot length, it is reduced by the factor .08. The model also shows that more rapid variations of air temperature are rapidly attenuated along the pipe. For the worst case

hot summer day when the air temperature is about 105F, 35F above the 70F temperature of the PVC pipe, this peak swing is reduced to 10F for a 50 foot pipe and 2.9F for a 100 foot length.

The specification for what swing is allowable for the air arriving at each antenna comes from the requirement that the sensitivity of on-the-fly continuum observations with the individual antennas not be limited by receiver gain variations resulting from receiver physical temperature variations. The air temperature variations are sinusoidal and have their maximum change near sun-up and sun-down. The receiver temperature regulating system is expected to have a gain (reduction factor) of about 10 and use the incoming air as its reservoir. With this amount of temperature stabilization, the residual fluctuation for the 50 length of pipe, 10F, is acceptable. For the 100 foot length, there is considerable margin. About 20% of the runs for the ATA-32 are less than 100 feet. It would suffice to use just the remaining 80% of antennas for the single dish mapping. There appears to be considerable margin. Our recommendation is that the node simply provide ambient air blown into the pipes to the antennas. No heating, air conditioning, or water reservoir is needed, in agreement with one suggested option in the DAVCO report.

1 Introduction

This is an extension of the very useful DAVCO report on the ATA node air conditioning options. The main new items are the choice of slightly different thermal properties for the soil, a narrowing of the temperature regulation specifications, and some slightly more analytic treatment of the heat diffusion problem. The first new item is a calculation of the penetration of heat into the ground associated with the daily and annual surface temperature variation. The second item is the formal discussion of the variation of the air temperature along the underground pipe associated with the air flow. A part of this consideration is the diffusion of heat through the PVC pipe and the ground, including the effect of a sinusoidal time variation in the input temperature. Lastly, the result of the air temperature filtering calculation is compared with the required temperature regulation at the antenna.

2 The Thermal Penetration Depth of Surface Temperature variations

This is a classical calculation discussed by Carslaw and Jaeger and quoted in the DAVCO report along with temperature statistics for the site. Equally important is the typical time variation of the surface temperature. Figure 1 shows the run of the temperature during July '03 and January '04. Apart from the effect of weather which tends to smooth the variations, the time dependence is dominated by two sinusoids, the daily and the annual. Since the heat flow problem is linear, the two problems may be considered separately and added. The temperature distribution along the coordinate distance into the ground x for a periodic surface temperature T_s at frequency Ω is given by the relation

$$T(x,t) = T_s e^{-x/\delta_d} cos(\Omega t - x/\delta_d) \tag{1}$$

where the penetration depth, δ_d is given by

$$\delta_d = \sqrt{\frac{2k}{\Omega C_v}} \tag{2}$$

k is the soil conductivity, and c_v is the volume heat capacity. The key issue is what numbers to use for these. From the point of view of cooling of components and air conditioning in general, the most difficult time is mid summer afternoon when the air temperature may exceed 100°F. The rest of the year is much less of a problem. We use numbers for the deep summer when the soil is both dry and sandy. The heat capacity per mass is 0.2 cal/deg C/gm for a wide range of solids. We found examples of densities of sandy soil, and 1.8 gms/cc is typical. This gives a volume heat capacity of .38 cal/deg C/cc. For heat conductivity we found a plot for sandy mixtures of hydrates for a range of hydrate fraction. The average of the distribution is 0.7 watts/m-K. This also corresponds to the 20% fraction. Even lower numbers are possible for dryer soil. Putting these numbers into (2) along with the Ω for a one day period produces a depth of 11 cm (4.3 in). At a depth of 24 in (2 ft), the periodic temperature is reduced by the factor .0039. This result agrees with a study I made thirty years ago at Hat Creek when we were planning to bury coaxial cable at the site. The 3 - 4 foot number from Carslaw and Jaeger is appropriate for normal moist soils. Although the soil moisture content is higher in other times of the year, lower than normal values are still appropriate for the Hat Creek soil.

The choice of 24 inches for the burial depth of the PVC pipes means that, especially in summer, the pipes will be at close to the average daily temperature. The annual variation, being much slower, is felt much more deeply, as noted in the DAVCO report. The amplitude of that surface temperature component can be estimated from the average of the January and July temperatures in Figure 1. The July average is 23^{o}C (73.4F), and the January average is 6.5^{o}C (43.7F). The penetration depth of this component (for the same soil at depth) is greater by $\sqrt{365}$. At a depth of 24 inches, the annual variation gives us an average mean temperature of about 70F, about 5F lower than the average surface value. The phase shift for the large scale temperature variation at the 24 inch depth is about one half month, so the hottest temperature in the summer two feet down occurs during August.

3 Variation of the Air Temperature along the PVC Pipe

Consider first the variation of the air temperature in the pipe as a function of position along the pipe. Assuming that the radial thermal impedances of the air/wall interface in the pipe, the PVC pipe, and the earth are known and are constant along the pipe, we can write down a simple relation between the temperature and position along the pipe. Let x be the distance down the pipe. The DAVCO report discusses the various thermal resistances. The radial convective heat transfer at x for a length of pipe dx is (in British thermal units)

$$H = \frac{T_{air} - T_{wall}}{R_{air}} dx BTUH \tag{3}$$

where $R_{air} = 1/\mathrm{Ah}$, is the resistance of the air/wall interface. h is the Nusselt number, and A is a unit length area along the pipe. Associated with this loss of heat to the wall is a loss in heat in the airflow with a corresponding temperature change dT_{air}.

$$H = -(.87)(CFM) dT_{air} BTUH \tag{4}$$

CFM is the airflow in CFM. These two items can be equated. The total radial impedance to the distant ground is $R_T = R_{air} + R_{PVC} + R_e$. This should be constant along the line. Using the resistive divider analog, we can write

$$T_{air} - T_{wall} = (T_{air} - T_g)\frac{R_{air}}{R_T} \tag{5}$$

where T_g is the constant temperature of the distant soil. Let τ = T_{air} - T_g. Combining these various relations produces a simple differential equation.

$$\frac{d\tau}{dx} = -\frac{\tau}{.87(CFM)R_T} \tag{6}$$

The solution is

$$\tau(x) = \tau(0)e^{-\lambda x} \tag{7}$$

where λ = 1/.87(CFM)R_T. The values for the various quantities that make up the different thermal resistances can be adopted from the DAVCO report p.16. We derived similar values with a few, such as the thermal conductivity, somewhat different. With the choices of thermal conductivity and volume heat capacity discussed above, we find (in British thermal units) k = .406 BTUH/ft ^{F} and for thermal diffusivity α = .018 ft^2/hr. The earth log with inner diameter of 3.5 inches, outer diameter of 3 feet, and the above thermal conductivity has R_e = .91. For the convective heat transfer coefficient in the PVC pipe with a flow of 50 CFM and corresponding velocity of 5 m/sec, we found a Reynold's number of 18,000 and a Prandtl number of .68 (All quantities are at STP here) and finally a Nusselt number,h, of 4.28, very close to the value of 4.55 BTUH/ft^2 F found by another method in the report. For the resistance of the air we get a rather different number. R_{air} = 1/hA = .30, rather than the value .0175 in the report, which is probably a simple misprint. Adopting 0.22 as the resistance of the PVC pipe from the report, we find a total thermal resistance of 1.4 F ft/BTUH. This gives a value of λ of .016. This is all for steady state.

We can now work out what will happen when we blow ambient air through various lengths of pipe. For a 50 foot length of pipe the offset air temperature is reduced by a factor of .45 and for a 100 foot length a factor of .20. So we agree with the remark in the report that for longer lengths of pipe probably no other heating or cooling is needed. The air comes out of the end of the pipe at the mean temperature of the ground, 70^{o}F as noted above. In fact, this steady state result is not really relevant because, as shown in figure 1, the air temperature during any given day is very close to a sinusoid at the daily period. There is also another issue besides the cooling of the air that is delivered to the individual antennas. It is important that the temperature of the air not vary more than can be compensated by the temperature regulators of the receiver. We return to this issue later.

4 The Frequency Dependence of Tube Filtering Effect

We can use the same formula (6) for the variation of the air temperature along the tube, but this time we must use the effective impedance as a function of frequency of the various thermal "resistances". There is essentially no delay in the heat transfer to the wall with the air moving at about 16 ft/sec, as compared with the length of a day. So the same R_{air} is appropriate. However, the diffusion through the PVC wall and the ground are slow, with time scales comparable to the variation of the air temperature. So we must work out the time dependence of the thermal diffusion. The effect of the ground is substantially bigger than that of the PVC, so for a first cut we keep the PVC effect as a simple resistance and study the ground diffusion in detail. We assume that the diffusion through the ground depends only on radius and time. The diffusion equation is

$$\nabla^2 T = 1/\alpha \frac{\partial T}{\partial t} \tag{8}$$

The process is linear, and we can get the sinusoidal response by assuming a time dependence of the form $e^{-i\omega t}$ in the above equation, giving us a complex temperature distribution. The new equation is

$$\frac{d^2T}{dr^2} + \frac{dT}{rdr} + \beta^2 T = 0. \tag{9}$$

where $\beta^2 = \frac{i\omega}{\alpha}$. We can get the temperature as a function of time by writing T(r,t) = Re(T(r) $e^{-i\omega t}$). Re means that we take the real part.

Equation (9) is Bessel's equation, and the form of its solution is known.

$$T(r) = AJ_0(\beta r) + BY_0(\beta r) \tag{10}$$

We find the coefficients A and B by satisfying boundary conditions. At r_1 $T(r_1) = T_{PVC}$. At some very large radius, r_2 the temperature is close to the average ground temperature. $T(r_2) = T_g$. The heat flow from the surface of the PVC pipe into the ground is given by the temperature gradient at r_1.

$$H_0 = -kArea\frac{\partial T}{\partial r}|_{r_1} = k\beta Area(AJ_1(\beta r_1) + BY_1(\beta r_1)) \tag{11}$$

If r_2 is sufficiently large, H_0 is independent of the actual values of r_2 and T_g. The thermal impedance is given by the ratio

$$Z_e = T_{PVC}/H_0 = \frac{\sqrt{p}}{.9945(1+i)}\frac{\Delta}{Q} \tag{12}$$

where

$$\Delta = J_0\left[\frac{.39(1+i)}{\sqrt{p}}\right]Y_0\left[\frac{8.0(1+i)}{\sqrt{p}}\right] - J_0\left[\frac{8.0(1+i)}{\sqrt{p}}\right]Y_0\left[\frac{.39(1+i)}{\sqrt{p}}\right] \tag{13}$$

and

$$Q = Y_0\left[\frac{8.0(1+i)}{\sqrt{p}}\right]J_1\left[\frac{.39(1+i)}{\sqrt{p}}\right] - J_0\left[\frac{8.0(1+i)}{\sqrt{p}}\right]Y_1\left[\frac{.39(1+i)}{\sqrt{p}}\right] \tag{14}$$

p is the period of the sinusoidal heating of the ambient air temperature.

We can now work out quantitatively the reduction in the air temperature swing in the pipe as a function of the length of the pipe. We use the values for the parameters discussed above. For p=1, a one day period, Z_e = .3812 + i.2070, substantially different than the steady state value. For R_{air} = .30, R_{PVC} = .22, and a flow of 50 CFM, we find that $|\lambda|$ = .025. For a 50 foot length of pipe the amplitude of the sinusoidal air temperature swing is reduced by the factor .29. For 100 feet, the reduction is factor of .082. These results are summarized in the table which also shows the expected residual temperature modulation for the worst case summer day. On that day the air temperature is taken to be 105F, which is 35F above the average temperature at the depth of the pipe, 70F. Already it is clear that at a pipe length of 100 feet or more there is little need for heating or cooling at the node, in agreement with the suggestion of the DAVCO report. Recall that we did not make use of the fact that the PVC pipe will also produce some further filtering Thus, the above calculation should be conservative.

Pipe length	exp(-.025 l)	$\times$ 35F
50 ft	0.29	10.2 F
100 ft	0.082	2.9 F
200 ft	0.024	.84F

It is also useful to understand the frerquency response of the pipe filter. That is, what if sudden weather changes occur that quickly change the air temperature. There is some such evidence in the January data of Figure 1. $|Z_e(p)|$ is plotted in Figure 2. $|Z_e|$ drops rapidly at periods less than one day, indicating that rapid air temperature changes will be smoothed out in the pipe.

5 Are the Regulation Specifications met by the Residual Temperature Fluctuations?

The maximum allowable temperature variation is set by the maximum allowable gain variation in the front end receiver. This is dictated by the observation that requires the best gain stability, which is on-the-fly continuum mapping with the antennas as single antennas rather than interferometer elements. The on-the-fly procedure makes a raster scan across a radio source, taking data along the way, followed by a loop back for the next row taking data during the loop back interval. This latter interval serves as the "off" observation to which each "on" data point in the row is compared. The turn around has typically the same time extent as the total time covered in the row. Hence, the signal/noise in the comparison is dominated by that of the individual row measurements.

We consider the requirement for a continuum observation at 20 cm wavelength. The beam width is 2.5^o, and the scan rate is about 2.5^o /second. At 1/2 beam sampling, the Nyquist rate, the time on the source is 1/2 second. The noise in one measurement with a bandwidth B, sample time t_s, and system temperature T_{sys} is given by

$$\frac{\Delta T}{T_{sys}} = \frac{1}{\sqrt{Bt_s}} \tag{15}$$

For a bandwidth of 100MHz and sample time of 1/2 second, the above fraction is .00014. In order for gain fluctuations not to dominate this number, the fractional gain fluctuations must be less than this. Measurements on the PAM amplifier show that its fractional gain/temperature sensitivity is .002/F. For this not to dominate the thermal noise fluctuation, physical temperature fluctuations must be less than .00014/.002 by a factor of about 3, that is, .023 degrees F. Temperature/gain effects in the optical fiber driver also matter, and let's assume that the factor of 3 will include them. The other important factor is the time scale over which we must have this stability. For a $20^o \times 20^o$ map with a sweep of 10 seconds and a turnaround time of 10 seconds, that gives about a 10 second time between each pixel observation and the turnaround. This is the time scale over which we need that gain stability. The residual temperature of the air in the pipe at the antenna is oscillating with a one day period with an amplitude A., $T_{air} = \mathrm{A}sin[(2\pi)t/P]$, peaking at mid day. The maximum rate of change of this temperature occurs one quarter of a day before and after mid day, and the maximum temperature rate of change is A $2\pi/P$. The temperature change in a time interval of Δt seconds is A $2\pi/P \times \Delta$t. For the 50 foot length of pipe, we found A = 10F, which leaves us with a temperature change of .0007 Δt. For Δt = 300 seconds, 5 minutes, we expect a temperature change of a .21 F. The PAM is equipped with a temperature regulator which should provide a regulation gain of at least 10. This will bring the regulation of the PAM to the required level of .023 F. Five minutes is much longer than the 10 seconds needed for the on-the -fly comparison in our example. So the temperature regulation is no problem at all. The more important effect of the pipe filtering is to provide cooled air at the antenna to take away heat from the components at the antenna.

6 Conclusion

Even for the shortest pipe length, 50 feet, with its residual amplitude swing of about 10F, the filtering effect of the pipe combined with the PAM regulator appears to be more than adequate. Of course, there are uncertainties in the numbers and calculations. For the longer pipes, there is even more margin. The most serious requirements are for the single antenna on-the-fly observations, and these do not have to be made using all the antennas. Leaving out those antennas with pipes less than 100 feet in length would not seriously degrade the observations. The recommendation is therefore that the smoothing of the air temperature by the pipes is adequate and no heating or cooling or water is needed at the nodes. A fan with a dust filter blowing 600 CFM of ambient air into the node is the sole requirement.

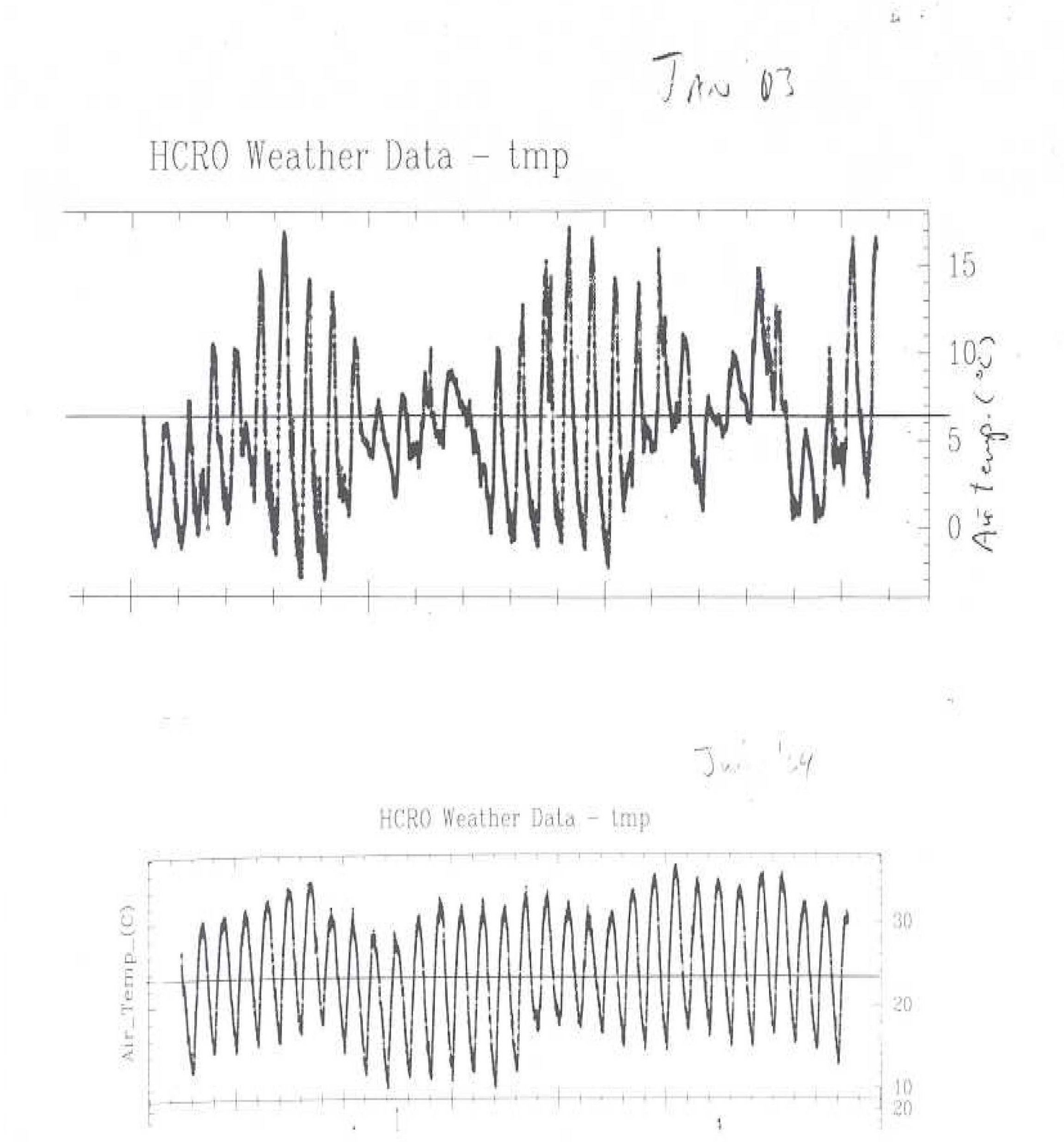

Figure 1: Air temperatures measured at Hat Creek over a month's interval. The top curve is for January of 2003. The bottom curve is for July of 2004. Temperatures are in degrees Celcius. The abscissa has one day tick marks for the bottom set and two days for the top.

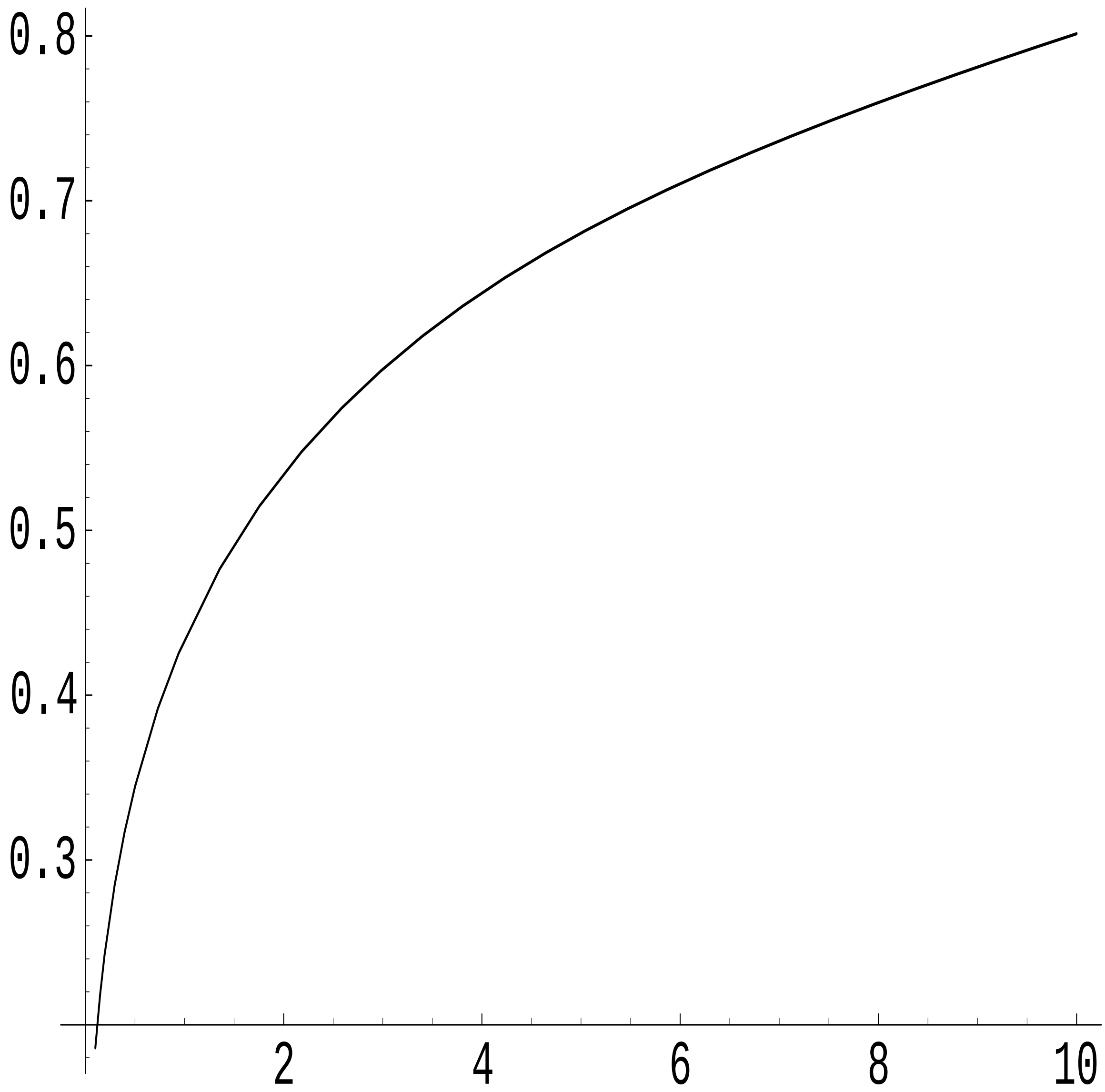

Figure 2: The magnitude of the thermal impedance of the ground around the the PVC pipe. The absiccsa is the sinusoidal period in days. The ordinate is the magnitude of the thermal impedance in F-ft/BTUH. At a period of one day, the impedance is $Z_e = .381 + i.207$.