

# Frequency Dependence of ATA Antenna Effective Area and Beamwidth with Focus Set for 6GHz

Jack Welch  
UC Berkeley

October 27, 2007

## 1 Effect of Axial Focus Error, $\Delta x$

The phase center of the feed operating at wavelength  $\lambda$  is at  $x = 1.4\lambda$ , where  $x$  is the distance along the feed axis from the feed vanishing point. For operation at a particular wavelength, the corresponding position  $x$  should be at the focal point of the optical system. For that choice of  $x$ , operation at any other wavelength with its different phase center will experience a quadratic phase error with peak value  $\Phi_m$  at the edge of the exit window 6m diameter aperture given by

$$\Phi_m = \frac{2\pi}{\lambda} \frac{2\Delta x}{1 + (4F)^2} \quad (1)$$

where  $\Delta x$  is the difference between where the antenna is focused and  $x(\lambda)$  referred to the focus position for 6 GHz, and  $F$  is the beam focal ratio, .65 for the ATA. If the antenna were radiating, the electric field at an angle  $\theta$  from the optical axis would be proportional to

$$\psi(\theta) \propto \int_0^3 e^{i\Phi_m(r/3)^2} [.210 + .786(1 - (r/3)^2)^{1.9}] J_0\left[\frac{2\pi}{\lambda} r \sin(\theta)\right] r dr, \quad (2)$$

where the term in the square braces is the aperture electric field produced by the feed and  $J_0$  is the Bessel function. The effective antenna collecting area is proportional to the square of the above integral for  $\theta = 0$ . It also

depends on feed losses, aperture illumination, spillover and surface roughness. The measured surface roughness is 0.7mm RMS, and the fractional aperture efficiency,  $E$ , including the above effects but without any focus error, is given in column 5 of the table below. The aperture efficiency exceeds 0.5 at all frequencies. The efficiency, including focus error,  $E(\text{of})$ , is in column 6. Column 7 shows the 3db beam size with no focus error. It is given by  $3.495(\lambda(\text{cm})/30)$  degrees. Column 8 shows the beam size including focus error.

Freq(GHz)	x(cm)	$ \Delta x _6$	$\Phi_m$	$E$	$E(\text{of})$	FWHM	FWHM(of)
0.5	86.4	79.2	2.14	.584	.415	$6.99^\circ$	$7.99^\circ$
1.0	43.2	36.0	1.95	.589	.444	$3.50^\circ$	$3.69^\circ$
1.5	28.8	21.6	1.75	.589	.470	$2.33^\circ$	$2.43^\circ$
2.0	21.6	14.4	1.55	.587	.491	$1.75^\circ$	$1.80^\circ$
4.0	10.8	3.6	.78	.574	.549	$.87^\circ$	$.88^\circ$
6.0	7.2	0.0	0.0	.559	.559	$.58^\circ$	$.58^\circ$
8.0	5.4	1.8	.78	.543	.519	$26.2'$	$26.3'$
10.0	4.3	2.9	1.56	.526	.440	$20.9'$	$21.6'$
11.5	3.7	3.5	2.15	.512	.364	$18.2'$	$19.4'$

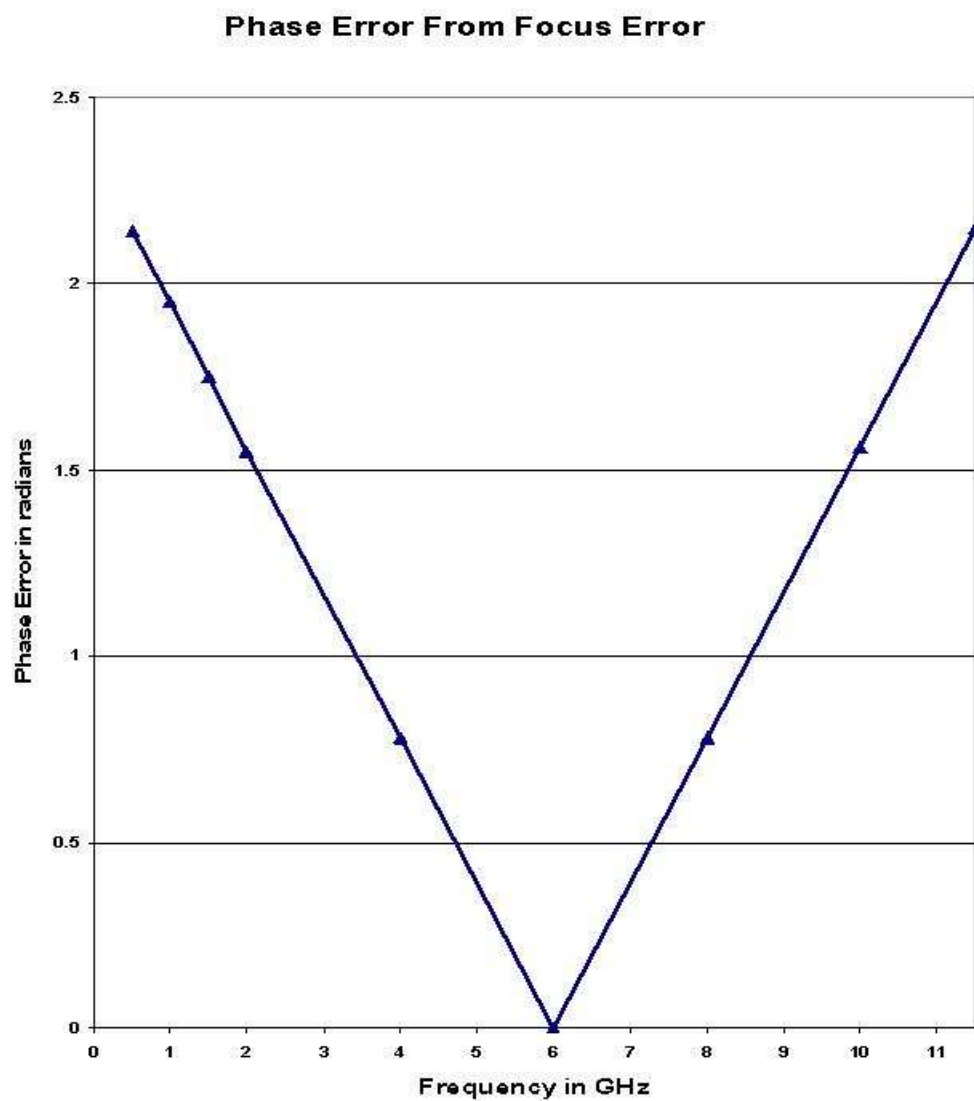


Figure 1: The edge quadratic phase error as a function of frequency for focus set at 6 GHz

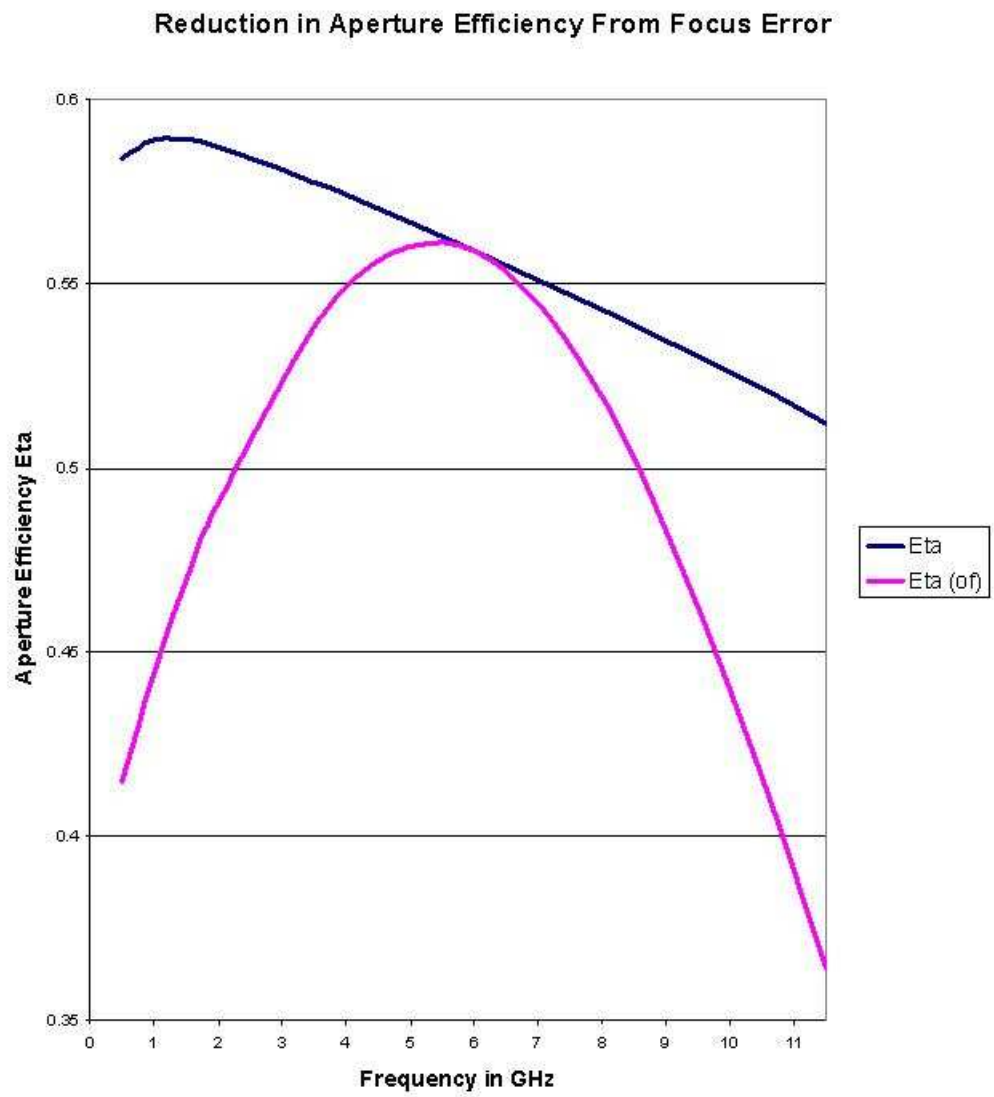


Figure 2: Beam efficiency as a function of frequency for focus set at 6 GHz

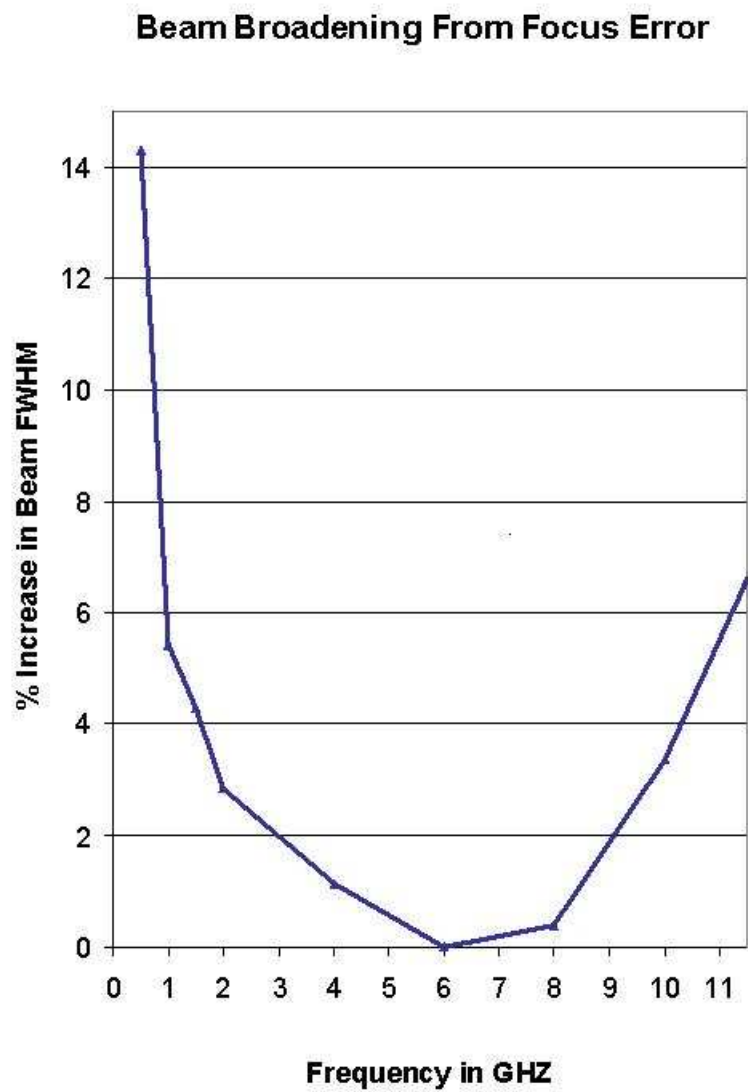


Figure 3: Beam broadening as a function of frequency for focus set at 6 GHz

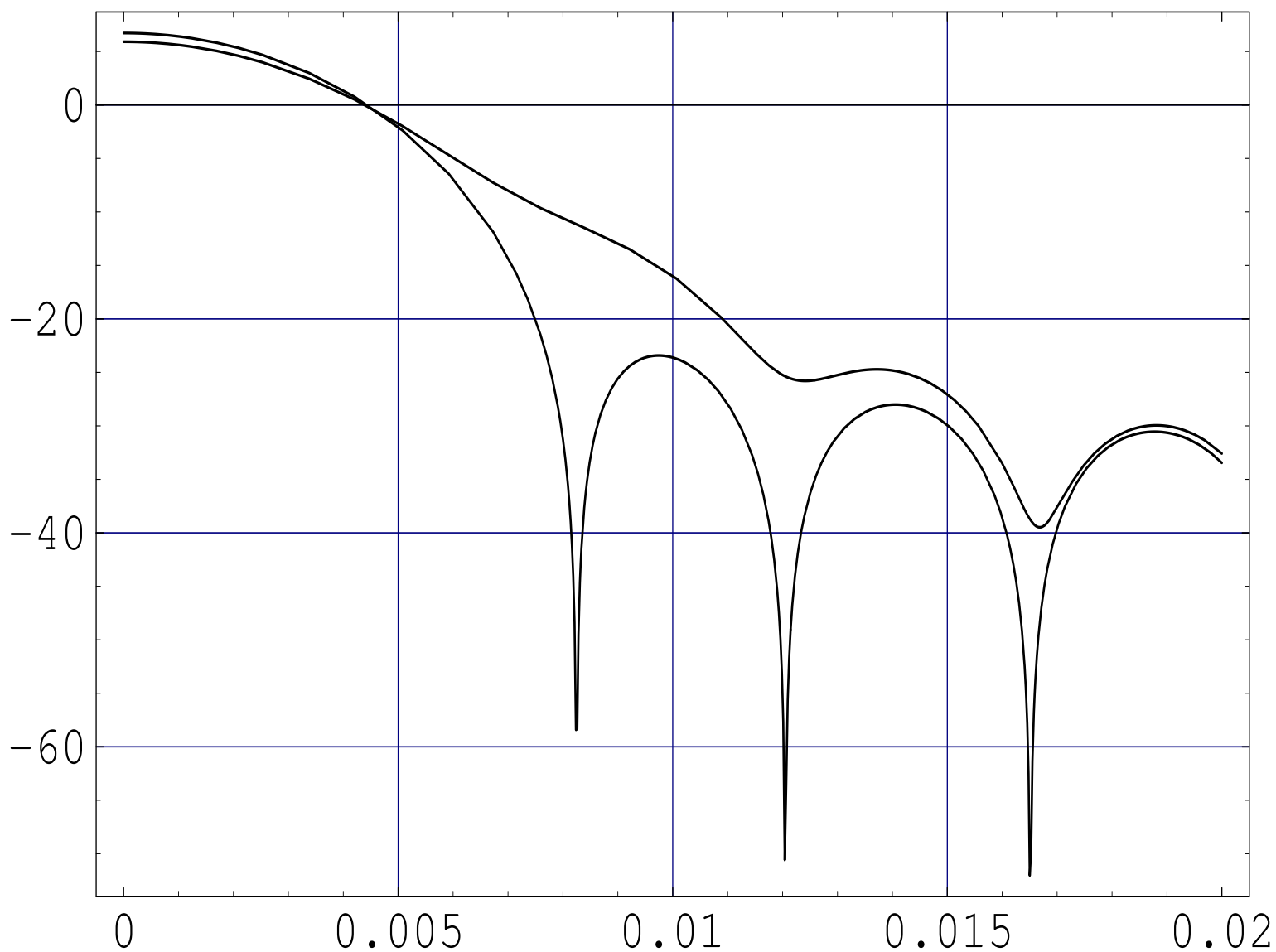


Figure 4: The pattern with sharp nulls is for the telescope focused at 10 GHz. The other pattern is also for 10 GHz but with the telescope focused at 6 GHz. The abscissa is angle in radians. The ordinate is in DB.